

AERO-OPTICAL PREDICTIONS OF HIGH-REYNOLDS NUMBER FLOWS
USING WALL-MODELED LARGE-EDDY SIMULATION

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by

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Abstract

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Reliable prediction of optical wavefront distortions induced by compressible turbulent flow surrounding an aperture is crucial to the development of airborne laser systems. Large-eddy simulation (LES) with a wall model provides a promising high-fidelity simulation method for high-Reynolds-number aero-optical flows by avoiding the severe near-wall resolution requirement. In this study, wall-modeled LES is employed to predict and analyze aero-optical distortions of subsonic and supersonic turbulent boundary layers, and subsonic and transonic flows over cylindrical turrets at high Reynolds numbers. The results are compared to experimental measurements and previous results from wall-resolved LES at reduced Reynolds numbers.

For turbulent boundary layers at Mach numbers from 0.5 to 4.3 and Reynolds numbers up to $Re_\theta = 6.9 \times 10^4$, reasonable agreement is obtained between numerical predictions and experimental data in terms of the magnitude and structures of optical distortions. The optical statistics are dominated by large-scale flow structures in the outer layer. The density correlations and pre-multiplied power spectra capture the features of large-scale motions whose length scales are comparable to the boundary layer thickness.

Subsonic and transonic flows over a cylindrical turret are simulated under experimental conditions. The subsonic flow aero-optical results show agreement with

the experimental data of Gordeyev et al. (AIAA Paper 2005-4657) and previous wall-resolved LES results. For the transonic flow, the simulation accurately predicts the fundamental frequency of the shock-induced pressure fluctuations and harmonics (Vorobiev et al., AIAA Paper 2014-2357). It is found that the piston and tilt components of the optical distortions are highly correlated with the shock-induced pressure oscillations, and the shock motion is clearly identified in the unsteady wavefront, which gives valuable input for the design of adaptive-optics systems synchronized with surface-pressure sensors. The LES data is also employed to provide a better understanding of the underlying physics of the shock-separation interaction.

This work is dedicated ...

to Sanaa & Sayed, my parents. No words can express my gratitude for all your sacrifices.

to Sara, my best friend & wife. This work cannot be done without your support & patience.

to Tamer & Wael, my brothers. Thanks for your encouragement.

to Malik, Omar & Ali. Thank you for being in my life

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SYMBOLS

a	Speed of sound
A_p	Aperture size
$A_{x'}$	Streamwise aperture length
$A_{z'}$	Spanwise aperture length
c	Speed of light
c_0	Speed of light in vacuum
C_f	Skin friction coefficient
C_0	Piston component of optical distortion
C_x	Tilt (streamwise jitter) component of optical distortion
C_z	Tip (spanwise jitter) component of optical distortion
C_{sc}	Shock-capturing coefficient
D	Diameter of the cylindrical turret
f	Frequency
k	Optical wavenumber
K_{GD}	Gladstone-Dale constant
L_{sponge}	Thickness of sponge layer
L_{top}	Thickness of a dissipative layer under the top boundary
M	Mach number
M_∞	Freestream Mach number
n	Index of refraction
N_x	Number of mesh points in the streamwise direction
N_y	Number of mesh points in the vertical direction

N_z	Number of mesh points in the spanwise direction
OPL	Optical path length
OPD	Optical path difference
p	Fluid pressure
P	Time-averaged pressure
P_{back}	Back pressure
P_{in}	Mean static inlet pressure
$P_{o,in}$	Mean total inlet pressure
$\mathcal{P}$	Fourier transform of pressure fluctuations time series
R	Radius of the cylindrical turret
$\mathcal{R}$	Correlation coefficient
$\mathcal{R}$	Gas constant
Re	Reynolds number
Re_θ	Reynolds number based on momentum thickness
Re_D	Reynolds number based on turret diameter
Re_R	Reynolds number based on turret radius
St	Strouhal number
SR	Strehl ratio
T	Fluid temperature or Time period
T_{stats}	Time period of statistics collection
T_{cycle}	Time period of a cycle
t	Time
Δt	Time step
$\mathbf{u}$	Velocity vector
U	Mean streamwise velocity
U_c	Convective speed of OPD
u, v, w	Streamwise, wall-normal and spanwise velocity components

y_m	Wall-layer thickness, wall-normal location of matching-point
x, y, z	Spatial coordinates
x', y', z'	Local coordinates of the optical beam grid
$\Delta x, \Delta y, \Delta z$	Mesh spacing in x , y and z directions
$\Delta x', \Delta y', \Delta z'$	Mesh spacing in x' , y' and z' directions
x_c, y_c, z_c	Coordinates of the aperture center
x_t, y_t	Coordinates of the cylindrical turret axle
x_{re}	Recycling station

Greek Letters

δ	Boundary-layer thickness
δ^*	Displacement thickness
ϵ	Very-small positive number
θ	Momentum thickness or elevation angle of the optical beam
λ	Wavelength
ω	Angular frequency
ρ	Fluid density
τ_w	Wall shear stress
σ_ϕ	Phase distortion
ϕ	Phase angle
Φ	Power spectral density
Φ^*	Normalized power spectral density

Superscripts

+ Variable in wall units

Subscripts

∞ Free-stream variable

comp Compressible

cycle Cycle of a periodic oscillation

in Inlet

inc Incompressible

max Maximum value

min Minimum value

o Stagnation isentropic condition

out Outlet

rms Root mean square

re Recycling station

VD Van-Driest transformed

Delimiters

$\langle \rangle$ Area averaging over an aperture

$\langle \rangle_z$ Spanwise averaging

— Time averaging

$\frown$ Phase averaging

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CHAPTER 1

BACKGROUND AND OBJECTIVES

1.1 Aero-Optical Distortions

In airborne laser systems, such as communication, targeting, imaging and directed energy systems, turrets are widely used to point and steer the laser beam. However the complicated flow features created by the bluff body of the turret can affect the performance of the optical systems. These features may include turbulent boundary layers, separated shear layers and wakes, as illustrated schematically by Gordeyev and Jumper [18] in Figure 1.1(a). At transonic flight speeds as the free-stream Mach number (M_∞) exceeds the critical limit (M_{critical}), local shocks are created over the surface of the turret, leading to an earlier flow separation and strong unsteady density variations, as shown in Figure 1.1(b) [18]. All of these compressible turbulent flow features lead to detrimental “*aero-optical*” distortions, causing beam defocus, jitter, and a dramatic reduction in its effective range.

“Aero-optics” is concerned with the aberration effects of compressible turbulent flows induced by solid surfaces near the aperture and the corresponding control and mitigation techniques [16, 31, 32, 69]. These optical distortions are caused by density fluctuations in the flow region. When a beam of an initially planar wavefront travels through a variable density field, as depicted in Figure 1.2, different parts of the beams propagate at different local speeds of light, which leads to distortions of the wavefront.

The propagation of electromagnetic waves through a compressible turbulent flow-field is governed by the Maxwell equations which are connected to the compressible

Navier-Stokes equations [45, 58, 69] because the index-of-refraction depends on the fluid density. However, since the timescale of optical propagation is much less than the flow timescale, the two equation sets can be decoupled in the solution process, and the flow-field can be assumed frozen when solving for the optical wave propagation at each time instant.

Furthermore, the optical wavelength (λ) is generally much smaller than the smallest turbulence length scale or the Kolomogrov scale. Consequently, the effect of depolarization can be neglected, and the Maxwell equations are reduced to a wave vector equation, where the three components are decoupled. Each scalar component, U , of

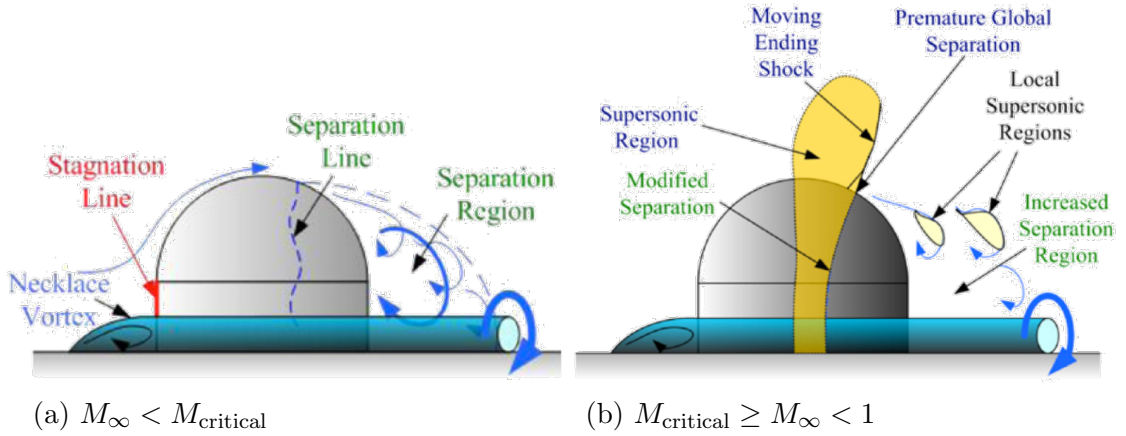


Figure 1.1. Flowfield around a hemisphere-over-cylinder turret (Gordeyev and Jumper [18]): (a) Subsonic flow speed; (b) Transonic flow speed.

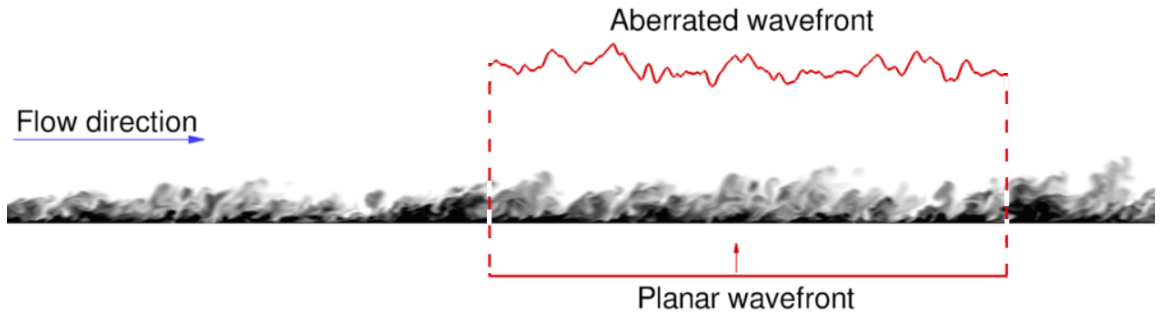


Figure 1.2. The aberrating effect of compressible turbulent flow on an initially planar optical wavefront traversing a boundary layer. Instantaneous density is shown in gray-scaled contours.

the electric field at frequency ω is governed by

$$\nabla^2 U + (k_0 n)^2 U = 0, \quad (1.1)$$

where $k_0 = \frac{2\pi}{\lambda} = \frac{\omega}{c_0}$ is the optical wavenumber. c_0 is the speed of light in vacuum, and n is the index of refraction. Since the fluctuations of the index of refraction are small, the optical beam propagates predominantly in the direction normal to the aperture, and the wave equation can be parabolized [45, 58, 69]. The solution to the paraxial wave equation across a relatively thin aero-optical region (from $y = 0$ to $y = L_y$) can be expressed as [45, 58, 69]:

$$U(x, y, z, t) = U(x, 0, z, t) \exp \left[-i \frac{2\pi}{\lambda} \int_0^{L_y} n(x, y, z, t) dy \right], \quad (1.2)$$

which indicates that the optical distortions are mainly phase distortions. The integrand in (1.2) is defined as the optical path length

$$\text{OPL}(x, L_y, z, t) = \int_0^{L_y} n(x, y, z, t) dy. \quad (1.3)$$

The index of refraction, n , is linearly related to the density of air through the Gladstone-Dale relation [17].

$$n(x, y, z, t) = 1 + K_{GD} \rho(x, y, z, t), \quad (1.4)$$

where K_{GD} is the Gladstone-Dale coefficient. It is in general weakly dependent on the optical wavelength but has a nearly constant value of $2.27 \times 10^{-4} \text{m}^3/\text{kg}$ within the visible wavelength range of 380 – 750 nm.

The wavefront distortions are represented by the relative difference in OPL over a beam cross-section. This is called the optical path difference (OPD), and defined

as

$$\text{OPD}(x, L_y, z, t) = \text{OPL}(x, L_y, z, t) - \langle \text{OPL}(x, L_y, z, t) \rangle, \quad (1.5)$$

where the angle brackets denote spatial averaging. The spatial root-mean-square of OPD is a function of time, i.e. $\text{OPD}_{rms}(t)$. Although its absolute value is quite small, it can be comparable to the optical wavelength. The optical phase difference

$$\sigma_\phi = \frac{2\pi \overline{\text{OPD}_{rms}}}{\lambda}, \quad (1.6)$$

where the overbar indicates time averaging, is typically of $\mathcal{O}(0.1)$ to $\mathcal{O}(1.0)$. For a given flow field, the magnitude of $\overline{\text{OPD}_{rms}}$ is fixed, and Eq.(1.6) shows that the phase distortion is inversely proportional to the optical wavelength, making the aero-optics problem more critical for short-wavelength airborne optical systems [31]. On the other hand, the short-wavelength laser systems are preferred in the absence of aero-optical aberrations because the diffraction-limited optical intensity with a fixed output power scales with $A_p^2/(L^2\lambda^2)$, where A_p is the aperture size, and L is the propagation distance.

Optical system performance is often measured in a crude way by the Strehl ratio ($\text{SR}(t)$), defined as the ratio of the maximum far-field irradiance $I(t)$ to the diffracted-limited on-axis irradiance I_0 . In the case the spatial probability distribution of $\text{OPD}(x, L_y, z, t)$, in the $x - z$ domain, is Gaussian at a given time instant t and the aperture size is significantly larger than the characteristic turbulence length scales, the time-averaged Strehl ratio is defined in terms of the optical phase distortion by [41, 56]

$$\overline{\text{SR}} = \frac{\overline{I}}{I_0} \approx \exp[-\sigma_\phi^2]. \quad (1.7)$$

This equation allows an estimate of the far-field optical irradiance based on near-field phase distortions.

1.2 Related Experimental Studies

Many experimental investigations have been carried out to analyze the physics of aero-optical phenomena, and to examine the effectiveness of control devices for mitigating the aero-optical effects of boundary layers, separated shear layers and wakes. Some of the related experimental studies in canonical flow configurations are discussed below.

1.2.1 Aero-optical Effects of Turbulent Boundary Layers

Since more than a decade ago, a series of experiments has been conducted to study the aberrating effects of subsonic boundary-layer flows on optical wavefronts. Gordeyev et al. [24] conducted subsonic boundary-layer experiments in different facilities over a range of free-stream Mach numbers, boundary-layer thicknesses and viewing angles. The optical distortions are measured using Malley probes and high-speed Shack-Hartmann wavefront sensors. The large-scale structures in the outer layer of the boundary layer were identified as the dominate contributor to the aero-optical distortions. The convective speed of the optical aberration measured directly by Malley probes was found to be approximately 82% of the free-stream velocity. There could be a potential relation between optical distortions in boundary layers and elongated low-speed packets of hairpin vortices whose convective speed, as found by Adrian et al. [1] and Tomkins and Adrian [59], has approximately similar value. These hairpin vortices are suggested to be the cause of the observed dependence of the optical beam distortions on the elevation angle [11, 24]: wavefront aberrations are larger when the beam is tilted downstream than those when it is tilted upstream. The experimental studies of aero-optical effects of turbulent boundary layers were extended to supersonic flow speeds by Gordeyev et al. [22, 25]. Convective speeds were found to monotonically increase with the free-stream Mach number.

Based on the linking equation [58], scaling laws were developed by Wyckham

and Smits [73] and Gordeyev et al. [22], to predict the time-averaged level of optical distortions in terms of $\overline{\text{OPD}}_{rms}$ and compared to experimental aero-optical results for both subsonic and supersonic boundary layers. The scaling law proposed by Gordeyev et al. [22], is written in terms of the free-stream Mach number M_∞ , free-stream density ρ_∞ , boundary layer thickness δ and local skin friction coefficient C_f :

$$\overline{\text{OPD}}_{rms} = K_{GD} \rho_\infty M_\infty^2 \sqrt{C_f} G(M_\infty), \quad (1.8)$$

where $G(M_\infty)$ is a function of the free-stream Mach number.

Gordeyev et al. [24] compared their experimental results to those of the numerical simulations by Wang and Wang [65] at $M_\infty = 0.5$ and $Re_\theta = 3550$, and White and Visbal [70] at $M_\infty = 1.3$ and $Re_\theta = 1426$. The effects of elevation angle and aperture size were shown to generally agree with the results of numerical simulations, especially those of Wang and Wang [65]. However, there is a quantitative discrepancy proposed to be due to the Reynolds number mismatch, since the experimental Re_θ is an order of magnitude higher than the numerical values.

1.2.2 Aero-optical Effects of Flow over Cylindrical Turrets

The flow over a cylindrical turret mounted between two flat platforms of different altitudes has been investigated extensively as it represents a simplified model of the realistic three-dimensional turrets due to the similarities between the flow structure over the apertures of hemispherical and cylindrical turrets.

Gordeyev et al. [19, 21] conducted a series of experiments to investigate the aero-optical aberrations caused by separated shear layers over the flat window of a cylindrical turret, in incoming Mach number range of 0.4 to 0.5. They utilized a Shack-Hartmann wavefront sensor to uncontrolled collect wavefronts, and Malley probes to obtain temporal information about the wavefront distortions over the window. Ve-

locity and turbulence intensity profiles were collected at seven streamwise locations over the turret. The main source of optical distortions is the vortical flows structures in the free shear layer separated from the upstream sharp edge of the flat window. To mitigate optical distortions, passive control devices consisting of a row of vertical pins placed upstream of the turret were explored. Pins of different diameters, spacings and heights were tested, and the most effective configuration was identified. The mitigation mechanism was found to be associated with a reduction of the turbulent fluctuation level and correlation length in the main shear layer due to the generation of a secondary shear layer, which is relatively weak.

Aero-optical distortions induced by incoming transonic flow over a cylindrical turret with a conformal window have been investigated recently by Gordeyev et al. [23] and Vorobiev et al. [63]. Spatially and temporally resolved wavefronts were collected using a high-speed Shack-Hartmann sensor and flow visualization was performed with a Schlieren system.

The experiments of Gordeyev et al. [23] were conducted for incoming Mach number ranging from 0.6 to 0.67. The incoming flow speed is raised by decreasing the back pressure controlled by a vacuum pump. When the incoming Mach number exceed 0.63 a localized supersonic flow region began to appear near the top of the cylinder and a shockwave observed to form. The shock first appears as a weak shock adjacent to the turret surface, and becomes stronger and oscillatory with increasing upstream Mach number until the flow is choked. As the back pressure decreases further, the shock strengthens further, becomes less unsteady and tilts more toward downstream. With this range of inlet flow speeds, the shadowgraphs showed different patterns of shockwave/separation interactions.

A shock wave creates steep pressure gradients and induces additional aero-optical distortions on an outgoing beam. In the case of a moving strong shock over the cylindrical turret, $\overline{\text{OPD}}_{rms}$ was found to be seven times the value of the fully subsonic

case. However, $\overline{\text{OPD}}_{rms}$ dropped to a level comparable to the fully subsonic case in the case of a stationary strong shock, because its steady lensing effect was removed by subtracting the time-averaged OPL component to calculate the unsteady OPD. Beside the baseline experiment, mitigation experiments were carried out by Gordeyev et al. [23] using several passive flow-control devices upstream of the turret including a vertically-placed porous screen along the span and a spanwise row of small pins of different diameters and spacings. These devices showed high effectiveness in mitigating the shock dynamics and strength and aero-optical distortions.

Vorobiev et al. [63] have recently conducted extensive experiments of the same flow configuration. Spatially-temporally resolved wavefront measurements with high-speed Shack-Hartmann sensor were synchronized with high-speed shadowgraph system and unsteady pressure measurements on the turret surface in a range of transonic inlet Mach numbers of 0.65 to 0.68. They proposed a simplified model to predict the essential features of the shock/separation interaction.

1.3 Previous Computational Studies

Computational Fluid Dynamics (CFD) became a significant player in aero-optics research only in the last decade with the rapid development of high-fidelity simulation techniques and the growth in computational resources. This is because an accurate prediction of aero-optical distortions requires the flow solver to capture a range of optically relevant temporal and spatial turbulent flow scales, which is not feasible with the traditional CFD methods based on Euler or Reynolds Averaged Navier-Stokes (RANS) equations.

1.3.1 Aero-optical Prediction Using Wall-Resolved Large-Eddy Simulation

Methods of high-fidelity simulations include Direct Numerical Simulation (DNS), Large-Eddy Simulation (LES), and hybrid LES/RANS approaches. DNS, the most

accurate and computationally most demanding approach was utilized by Truman and Lee [61] and Truman [60] to study aero-optics. However, they solved low Reynolds number incompressible flow equations. The density fluctuations were modeled as a passive scalar. Their results showed that the anisotropic flow structures, in turbulent shear flows, cause the optical distortions to be strongly dependent on the direction of propagation.

LES is computationally less expensive than DNS, as it resolves only the energy containing spatial and temporal flow scales. A subgrid scale (SGS) model is applied to account for the effect of the unresolved fluid motions. Mani et al. [42] showed through a theoretical analysis that an adequately resolved LES is capable of capturing the aero-optics of highly aberrating flows without the need for additional aero-optical subgrid scale modeling. In other words, aero-optics does not impose additional resolution requirements. Mani et al. [43] simulated the flow over a circular cylinder at Mach 0.4 and two Reynolds number $Re_D = 3900$ and $Re_D = 10000$, based on the cylinder diameter. They found that the optical distortions caused by the separated shear layer in the near wake strongly depend on the Reynolds number. This is because the cylinder flow involves laminar separation, and the instability and transition of the separated shear layer are highly sensitive to Reynolds number.

The aero-optical effects of subsonic flow over a cylindrical turret with a flat window, as in the experiment of Gordeyev et al. [21], were computed using LES by Wang and Wang [66]. The flow was simulated for Mach 0.5 at two Reynolds numbers, $Re_R = 4.410^4$ and $Re_R = 8.8 \times 10^4$, based on the turret radius and free-stream velocity. The Reynolds numbers of these simulations are 8.2% and 16.4% of the experimental value. The computed $\overline{OPD}_{rms}$ agrees well with the experimental value of Gordeyev et al. [19, 21]. Based on the results of two simulations, they concluded that the optical distortions due to separated shear layers originating from turbulent boundary layers are insensitive to the Reynolds number, although separation effects

differ for laminar and turbulent boundary layers.

Aero-optical distortions caused by subsonic turbulent boundary layers were numerically studied using LES by Wang and Wang [65] for $M_\infty = 0.5$, at different Reynolds numbers $Re_\theta = 875, 1770$ and 3550 . Their results show that magnitude and spatial structures of wavefront distortions are dependent on Reynolds number in the range of Reynolds numbers considered. The time-averaged OPD_{rms} shows a decrease as the Reynolds number increases, but the rate of the decrease drops at higher Reynolds numbers. They also found that the spanwise correlation length of OPD decreases with Reynolds number, while its streamwise correlation length increases with Reynolds number. The contributions of different boundary-layer zones to the wavefront distortions were investigated, and it was shown that the dominate effects comes from the log-layer and wake region, while the contributions of the viscous sub-layer and buffer zone are small. Their results also confirmed the directionality of the optical distortions and its relation to the anisotropic turbulence structures.

The simulation conducted by Wang and Wang [65] at the highest Reynolds number $Re_\theta = 3550$ is computationally expensive. The number of the grid cells is approximately 50 million, and the time step size is rather small. However, this Reynolds number is still much lower than that of the experimental case ($Re_\theta \geq 1.0 \times 10^4$) used for comparison. Based on their analysis of contributions from various flow regions and flow scales, Wang and Wang [65] suggested that wall-modeled LES can be a promising method to simulate boundary-layer flows at high Reynolds numbers and investigate their aero-optical aberrations.

White and Visbal [71, 72] investigated the shock/boundary-layer interaction and the related aero-optical effects. The supersonic flow of $M_\infty = 2$ was simulated coming into a 20 degree compression ramp at two different low Reynolds numbers, $Re_\theta = 1374$ and 2263 . The results showed a remarkable difference between the two cases in the flow features and wavefront distortions behind the shock.

1.3.2 Aero-optical Predictions Using Hybrid LES/RANS Approaches

Practical aero-optics problems generally involve 3-D geometry and high Reynolds number. They are beyond the reach of LES with today's computational power. Hybrid LES/RANS approaches, such as detached-eddy simulation (DES) and its variations, therefore gained attention. Nahrstedt et al. [49] applied the partially averaged Navier-Stokes (PANS) method with the two-equations $k - \epsilon$ model to simulate the aero-optical flow over the hemisphere-on-cylinder turret as in the experiment conducted by Gordeyev et al. [20]. Ladd et al. [38] studied the same turret configuration using DES with the two-equations $k - \omega$ turbulence model. Morgan and Visbal [46, 47] employed a hybrid RANS/ILES approach with $k - \epsilon$ model to simulate the same experiment as well as another hemisphere-on-cylinder turret configuration. These hybrid simulations capture some of the global flow features such as the mean surface pressure and wake profiles at some locations, but overall, the agreement with experimental data is not satisfying in terms of both flow and optical results.

1.4 Wall-Modeled Large-Eddy Simulation

Wall-modeled LES [34, 55, 68] is another class of hybrid LES/RANS methods. Although it is computationally more demanding than standard DES methods, it is more accurate and robust [55]. In this method, the flow is computed using LES except in the near-wall region, which is generally “*modeled*” using a RANS approach. The computational expenses of wall-modeled LES is weakly dependant on the Reynolds number compared to wall-resolved LES. Choi and Moin [10] revisited the resolution requirements for LES estimated by Chapman [9]. They found that the total number of grid points N required for wall-modeled LES is proportional to Re_{L_x} , where L_x is the flat-plate length in the streamwise direction. On the other hand, wall-resolved LES demands that $N \sim Re_{L_x}^{13/7}$.

The most popular approach in wall-modeled LES is the two-layers zonal approach [55]. This approach employs two overlapping computational domains: the outer LES domain and near-wall domain. The two domains are coupled through two interfaces: the wall boundary and the matching interface at the top of the wall-layer mesh. The wall-normal height y_m of this secondary mesh typically does not exceed 5 – 10% of the local boundary-layer thickness. Due to the employment of approximate wall-boundary conditions, the LES mesh only needs to resolve the outer flow scales and not the near-wall small-scale structures. The first off-wall mesh spacing in the LES mesh can be orders of magnitude larger than the viscous wall unit, and accordingly the time step size will also be significantly enlarged compared to that of a wall-resolved LES using the same time-marching scheme. As a result, there is a dramatic reduction in computational expenses for wall-bounded flows compared to wall-resolved LES.

In the wall-layer domain, a simplified set of equations is used. As previously mentioned, the LES time step size is based on the outer flow scales which are much larger than the inner scales. Navier-Stokes equations are time- or ensemble-averaged inside the wall-layer domain, or in other words the Reynolds-averaged Navier-Stokes (RANS) equations are employed in the near-wall region. Based on the wall-model assumptions, a boundary-value problem is solved inside the wall-layer mesh.

The classification of different methods of zonal wall-modeled LES depends on the kind of approximate boundary conditions and the degree of fidelity of the wall model. There are two types of approximate boundary conditions [55]: the wall-shear-stress approach, and the off-wall boundary approach

The wall-shear-stress approach is more popular and widely used because it ensures well-posedness of the mathematical problem. In this method, the wall shear stress obtained from the wall model is applied as a Neumann boundary condition to the momentum equations in the wall cells in the LES. In the second approach, the wall

model provides LES with instantaneous slip velocities at an off-wall boundary of the LES mesh. It is attractive for certain discretization methods such as nodal-based finite-volume schemes, but there are several challenges concerning its robustness and accuracy as discussed by Cabot and Moin [8].

Wall-model approaches can also be categorized based on the complexity level of the RANS-type equations applied in the wall layer. The simplest set of equations is the equilibrium stress-balance model [8, 34], where one-dimensional ordinary-differential equations are solved across the wall layer as boundary-value problem. At the most complex level, fully three-dimensional RANS equations are solved in the wall layer. When a RANS-type eddy viscosity is used in the wall-layer equations with nonlinear convective terms, its value must be reduced to account for only the unresolved part of the Reynolds stress as discussed by Wang and Moin [68] and Cabot and Moin [8]. Wang and Moin [68] used a dynamically adjusted mixing-length eddy viscosity in the wall model by matching the Reynolds stresses from the LES and the wall model at the top boundary of wall-layer mesh, i.e. $y = y_m$.

Kawai and Larsson [35] adopted a spatially-variable dynamic approach for the parameters used in the mixing length eddy viscosity/diffusivity models for compressible flows. They assumed constant values for the wall-model coefficient ($\kappa_{wm} = 0.41$) and turbulent Prandtl number ($Pr_{twm} = 0.9$) for $0 \leq y \leq \alpha y_m$, where y is the wall-normal distance and α is fraction of the wall-layer thickness, and applied the dynamic matching as in [68] for κ_{wm} and Pr_{twm} at the matching point $y = y_m$. For $\alpha y_m < y \leq y_m$, κ_{wm} and Pr_{twm} were assumed to vary linearly. By increasing the eddy viscosity/diffusivity in the regime close to the wall, they obtained improvement in the wall-modeled LES results compared to those of Wang and Moin [68] for equilibrium boundary layers and shock/boundary-layer interaction.

Park and Moin [53, 54] developed a nonequilibrium wall model based on unsteady 3-D RANS equations implemented in a wall-layer unstructured mesh. Unlike the

aforementioned non-equilibrium models, the eddy viscosity and conductivity models do not involve a stress-matching procedure or a tunable free parameter, and the model shows consistent performance over a wide range of Reynolds numbers. Park [51, 52] used the nonequilibrium wall model to compute the separated flow over the NASA wall-mounted hump, and found that calculations with the equilibrium and nonequilibrium wall models produced nearly identical results. He suggested, however, that the difference in the model performance would be more pronounced for flows where history effects are important and non-equilibrium effects are global, as in the cases of stalled diffusers and massive separation on airfoils. These results of Park’s calculations indicate that the equilibrium wall-modeled LES can provide fairly acceptable results for a wide range of flow configurations.

Bermejo-Moreno et al. [4] assessed the ability of LES with an equilibrium wall model as presented by Kawai and Larsson [34] for predicting shockwave/turbulent-boundary-layer interaction (STBLI), which involves strong adverse pressure gradients and secondary flows, causing the boundary layer to be out of equilibrium. They found reasonable agreement between simulations and experiments, both qualitatively and quantitatively. Their results suggested that non-equilibrium effects, which were neglected in the near-wall model applied in less than 10% of the boundary layer thickness, are mostly relevant in the outer layer of the turbulent boundary layer, and therefore can be accurately captured by the LES and enter the wall model calculation through the top boundary conditions at the matching locations.

By using a compressible unstructured-mesh LES code, CharLES, developed by Cascade Technologies Inc. [36], Mathews et al. [44] recently employed the equilibrium stress-balance wall model to compute separated flow over a 3-D optical turret similar to the flow configurations which were studied earlier using DES-type approaches as mentioned previously. Their results provided the closest aero-optical predictions to the experimental measurement at high Reynolds numbers.

1.5 Objectives of the Present Study

The objectives of the present research are to use wall-modeled LES to predict the aero-optical distortions of turbulent boundary layers, and separated shear layers over a wide range of Mach numbers and Reynolds numbers, evaluate the accuracy of this approach, and characterize the distortions in terms of important parameters. The numerical results are compared to experimental measurements and previous wall-resolved LES results.

Both subsonic and supersonic turbulent boundary layer are considered at different free-stream Mach numbers over a wide range of Reynolds numbers. Before computing the optical distortions, it is essential to have validated predictions of levels of the fluctuations of velocities, density and index-of-refraction fields. A mesh convergence study was conducted in order to provide grid-insensitive aero-optical predictions. The numerical predictions are validated by the experimental results of Gordeyev et al. [22, 24, 25] for subsonic and supersonic boundary layers and wall-resolved LES results of Wang and Wang [65] for subsonic boundary layer at a relatively low Reynolds number. The relationship between the wavefront spatial structures and density-field structures inside the boundary layer was discussed.

Wall-modeled LES was also utilized to compute the subsonic flow over a cylindrical turret with a flat window, the same flow configuration as in the experiment conducted by Gordeyev et al. [21]. The results are compared with the corresponding experimental measurements and previous numerical results obtained by Wang and Wang [66] using wall-resolved LES at a reduced Reynolds number.

Finally, wall-modeled LES was employed to compute the aero-optics of transonic flow over a cylindrical turret with a conformal window, as in the experiment of Vorobiev et al. [63]. The emphasis is in capturing the low-frequency shock-separation oscillations.

A systematic procedure was followed to ensure that the numerical simulation was

conducted at the experimental conditions for a choked flow case with strong shock oscillations. The relationship between optical distortions and the shock oscillation above the optical aperture was investigated.

CHAPTER 2

METHODOLOGY

Compressible turbulent flow-fields are spatially and temporally resolved using wall-modeled LES. Based on the simulated fluctuating density field in the vicinity of the optical aperture, the instantaneous field of the index of refraction is calculated based on the Gladstone-Dale relation [17] as described in Eq.(1.4). The optical wavefront distortions are calculated by ray tracing using Eqs.(1.3) and (1.5).

2.1 LES Governing Equations

The governing equation for LES are the spatially-filtered Navier-Stocks equations along with the continuity and energy equations, which are expressed in the following dimensionless form:

$$\begin{aligned}
 \frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i}{\partial x_i} &= 0, \\
 \frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{u}_i \tilde{u}_j + \delta_{ij} \bar{p}) &= \frac{\partial \tilde{\tau}_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}^{SGS}}{\partial x_j}, \\
 \frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial}{\partial x_j} [(\bar{\rho} \tilde{E} + \bar{p}) \tilde{u}_j] &= \frac{\partial}{\partial x_k} (\tilde{u}_j \tilde{\tau}_{jk}) + \frac{\partial \tilde{q}_k}{\partial x_k} - \frac{\partial q_k^{SGS}}{\partial x_k},
 \end{aligned} \tag{2.1}$$

where ρ , u , p , E , τ , q , t and x are density, velocity, pressure, energy, viscous and subgrid-scale stress, heat flux, time and spacial coordinates, respectively. The indices i, j and k indicate the three spatial dimensions, and the overbar and tilde denote spatial and Favre filtering, respectively, to be defined later. The flow variables are normalized using a reference length L_{ref} , a reference density ρ_{ref} , a reference tem-

perature T_{ref} and a reference dynamic viscosity μ_{ref} . The reference velocity u_{ref} is taken to be the reference speed of sound $a_{ref} = \sqrt{\gamma p_{ref}/\rho_{ref}}$, where γ is the specific heat ratio ($= 1.4$). The energy is $\tilde{E} = \bar{p}/(\gamma - 1) + (\rho \tilde{u}_k \tilde{u}_k)/2$, and the dimensionless equation of state, $\bar{p} = \bar{\rho} \tilde{T}/\gamma$, is used for a perfect gas.

The spatial filtering of a function $f(\mathbf{x})$ over a domain Ω is expressed as

$$\bar{f}(\mathbf{x}) = \int_{\Omega} G_{\Delta}(\mathbf{x} - \mathbf{x}') f(\mathbf{x}') d\mathbf{x}', \quad (2.2)$$

where G_{Δ} is a spatial filter of width Δ . The Favre filtering is a density weighted filtering operation defined as

$$\tilde{f} = \frac{\bar{\rho} f}{\bar{\rho}}. \quad (2.3)$$

The resolved viscous stress and heat flux are

$$\tilde{\tau}_{ij} = \frac{\tilde{\mu}}{Re_{ref}} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{2}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right), \quad (2.4)$$

$$\tilde{q}_i = \frac{\tilde{\mu}}{(\gamma - 1)Pr} \frac{\partial \tilde{T}}{\partial x_i}, \quad (2.5)$$

where $Re_{ref} = u_{ref} \rho_{ref} L_{ref} / \mu_{ref}$ is the reference Reynolds number and Pr is the Prandtl number, which is set to 0.7. The variation of the dynamic viscosity $\tilde{\mu}$ with temperature is assumed to obey Sutherland's law, $\tilde{\mu} = (\tilde{T})^n$, where $n = 0.76$.

The subgrid-scale stress and heat flux represent the effects of unresolved fluid motions. They are defined as:

$$\tau_{ij}^{SGS} = \bar{\rho} (\widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j), \quad (2.6)$$

$$q_i^{SGS} = \bar{\rho} (\widetilde{u_i T} - \tilde{u}_i \tilde{T}). \quad (2.7)$$

2.2 Wall-Layer Model

In a wall-modeled LES, computational expenses are drastically reduced by imposing approximate boundary conditions on the wall instead of resolving the wall-layer with the conventional Dirichlet boundary conditions (e.g. no-slip condition for the momentum equations and isothermal condition for the energy equation). These approximate boundary conditions are obtained by solving a simplified form of Navier-Stokes equations in a Reynolds-averaged sense. In the present study, the equilibrium stress-balance model of the form [34]

$$\begin{aligned} \frac{d}{dy} \left[(\mu + \mu_t) \frac{du}{dy} \right] &= 0, \\ \frac{d}{dy} \left[(\mu + \mu_t) u \frac{du}{dy} + \frac{\gamma \mathcal{R}}{\gamma - 1} \left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{dT}{dy} \right] &= 0, \end{aligned} \tag{2.8}$$

are used. These equations are a simplified version of turbulent boundary-layer equations [8, 55, 68]. Variables x and y represent local spatial coordinates, with y in the wall-normal direction and x in the wall-parallel direction aligned with the tangential velocity component at the matching point $y = y_m$, and $\mathcal{R}$ is the gas constant. The turbulent eddy viscosity coefficient μ_t is provided by the mixing-length model

$$\mu_t = \kappa y^+ (1 - e^{-\frac{y^+}{A}})^2, \tag{2.9}$$

where $y^+ = y u_\tau \rho / \mu$ is the distance to the wall in wall units, $\kappa = 0.41$ and $A = 17$. The turbulent Prandtl number is set be constant, $Pr_t = 0.9$.

From numerical solutions of Eqs.(2.8), the wall shear stress and heat flux are

calculated based on

$$\tau_w = \mu \left. \frac{du}{dy} \right|_{y=0}, \quad q_w = \begin{cases} 0, & \text{adiabatic wall} \\ \frac{\gamma \mathcal{R}}{\gamma - 1} \frac{\mu}{Pr} \left. \frac{dT}{dy} \right|_{y=0}, & \text{isothermal wall} \end{cases} \quad (2.10)$$

which are used in the LES as approximate wall-boundary conditions.

2.3 Numerical Method and Solution Procedure

The wall-model equations (2.8) are solved on a wall-layer mesh which has the same resolution as the LES mesh in the wall-parallel directions but is refined to resolve the viscous length scale in the wall-normal direction, as shown schematically in Figure 2.1. An iterative method is employed to solve this boundary-value problem at each LES time step. The boundary conditions are no-slip and adiabatic (or isothermal) conditions imposed at the wall and Dirichlet conditions matching the instantaneous LES field on the top boundary. In this study, the typical matching point location is set to be at 5% of the boundary-layer thickness, i.e. $y_m = 0.05\delta$, which is within the logarithmic layer. The wall layer includes five equally-spaced LES cells in the wall-normal direction, with a wall-normal mesh spacing of $\Delta y_w = 0.01\delta$. As suggested by Kawai and Larsson [34], it is important to place the matching point in the well-resolved LES region, which requires a minimum number ($\gtrsim 5$) of LES grid points from the wall to resolve the energetic near-wall eddies whose size scales with distance to the wall.

The LES code ‘CharLES’, developed by Cascade Technologies Inc. [36], is used in the current research. The numerical scheme is based on a finite volume discretization of the governing equations

$$\frac{\partial \mathbf{Q}}{\partial t} V_{cv} + \sum_{faces} \{ \mathbf{F}_c + \mathbf{F}_d \} \cdot \mathbf{A}_f = 0, \quad (2.11)$$

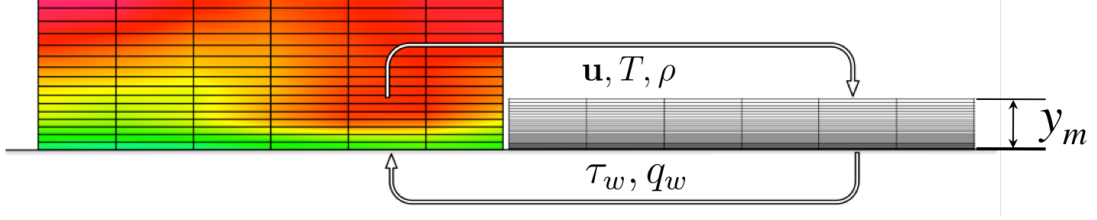


Figure 2.1. A schematic showing data exchange between LES mesh and wall-model.

where $\mathbf{Q} = \{\bar{\rho}, \bar{\rho} \tilde{u}_i, \bar{\rho} \tilde{E}\}^T$ is the vector of the flow variables. $\mathbf{F}_c$ and $\mathbf{F}_d$ are the convective and diffusive vector fluxes, respectively. V_{cv} is the volume of the mesh cell and $\mathbf{A}_f$ is the vector normal to a cell face with its magnitude equal to its surface area. Time marching is fully explicit based on a third-order Runge-Kutta method.

The convective flux at each cell face is computed in two steps: In the first step, polynomials for the primitive flow variables ρ , p and u_i are constructed based on the cells located at the right and left sides of the cell face centroid. The spatial accuracy of these polynomials depends on the geometry of the cells and is third-order at maximum for a uniform Cartesian mesh. In the second step, the flux at the face is computed using a blend of non-dissipative centered flux and a dissipative upwind-biased HLLC Riemann flux:

$$\mathbf{F}_c = (1 - \alpha)\mathbf{F}_{central} + \alpha\mathbf{F}_{upwind}, \quad (2.12)$$

where $0 \leq \alpha \leq 1$ is a blending parameter. The value of α is calculated in the preprocessing stage, based on the local mesh quality. The effect of the dissipative term becomes significant only in regions of relatively poor grid quality to avoid numerical instability. A dynamic version of Vreman's model [64] is applied to model the physical effects of the unresolved turbulence.

CharLES also has the capability of capturing shock waves using a hybrid central/essentially-

non-oscillatory (Central/ENO) scheme for the convective fluxes. It consists of three major components: First the central fluxes are computed and blended with dissipative upwind fluxes, whose fraction depends on the local mesh quality. Then the flow smoothness across the cell-face is examined using a local shock sensor based on the reconstructed face values and the upwind values on the right- and left-hand sides. If the shock sensor activates the hybrid switch, CharLES uses a second-order ENO scheme to perform the left- and right-biased face-reconstructions.

A more thorough description of the numerical techniques used in CharLES can be found in Khalighi et al. [36]

2.4 Method for Aero-Optical Calculations

As introduced in Section 1.1, the instantaneous index-of-refraction field has to be calculated in the region of the beam propagation through the turbulent flow-field. To facilitate the optical calculations a beam grid, of the same resolution as the LES mesh, is constructed along the beam path as illustrated in Figure 2.2. The instantaneous density values on the optical grid are interpolated from the LES grid using a second-order gradient-based method:

$$\rho_{optical} = \rho_{cv} + (\mathbf{x}_{optical} - \mathbf{x}_{cv}) \cdot \nabla \rho|_{cv}, \quad (2.13)$$

where $\rho_{optical}$ is the density value at an optical grid point and ρ_{cv} is the density at the centroid of the control volume containing the optical grid point. $\mathbf{x}_{optical}$ and $\mathbf{x}_{cv}$ are the position vectors of the optical grid point and the centroid of the LES grid cell, respectively. $\nabla \rho$ is the density gradient of second-order accuracy. The index of refraction is then determined by the Gladstone-Dale relation [17] shown in Eq.(1.4), and the OPL is calculated based on Eq.(1.3) by integrating along the optical path numerically.

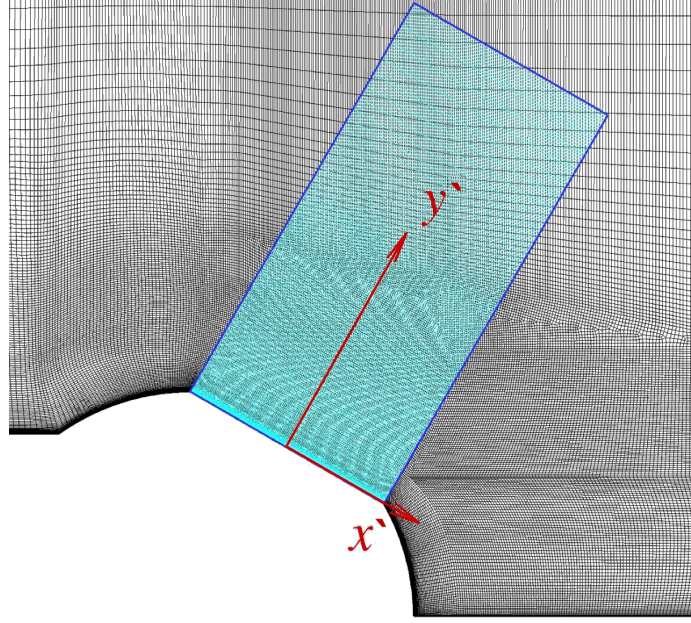


Figure 2.2. Schematic of the beam grid and LES grid.

In practice, the spatially linear components of optical distortions are removed in the OPD calculations,

$$\text{OPD}(x', z', t) = \text{OPL}(x', z', t) - [C_x(t)x' + C_z(t)z' + C_o(t)], \quad (2.14)$$

where x' and z' are the local streamwise and spanwise coordinates of the optical aperture, respectively. $C_x(t)$, $C_z(t)$ and $C_o(t)$ are known as the tip, tilt and piston components, respectively. They are calculated using a least-square method by minimizing the function

$$G = \iint_{A_p} \{\text{OPL}(x', z', t) - [C_x(t)x' + C_z(t)z' + C_o(t)]\}^2 dx' dz'. \quad (2.15)$$

CHAPTER 3

TURBULENT BOUNDARY LAYERS

In this chapter, prediction of optical aberrations caused by turbulent boundary-layers is discussed. Wall-modeled LES was utilized to simulate turbulent boundary-layer flows at subsonic and supersonic speeds ($M_\infty = 0.5, 2.0, 3.0$ and 4.3) over a wide range of Reynolds numbers from $Re_\theta = 3.55 \times 10^3$ to 6.9×10^4 . For all of these simulations, the mesh resolution is defined in terms of outer flow length scales thanks to wall-modeled LES which is liberated from the Reynolds-number-dependent near-wall mesh requirements. To determine the mesh requirement for flow and aero-optical predictions, a mesh convergence study was conducted for two flow cases at different Mach and Reynolds numbers: $M_\infty = 0.5, Re_\theta = 3.55 \times 10^3$ and $M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$. The simulation data were employed to determine the levels of density fluctuations and optical distortions at different Mach numbers and practical values of Reynolds numbers. The relation between the structures of the density fluctuations and optical wavefronts is discussed. Finally, the large- and very-large-scale motions [3, 37] are investigated based on the computed density and streamwise velocity fields on the most refined mesh. This work was partially presented in [33].

3.1 Flow Simulation Set-up

Five turbulent boundary layers were considered in this study as listed in Table 3.1. The first four cases match the flow conditions of the experiments conducted by Gordeyev et al. [22, 24, 25]. The fifth case has the same flow conditions of the numerical simulation of Wang and Wang [65] using wall-resolved LES. A schematic of the computational setup to simulate turbulent boundary layers using wall-modeled LES is shown in Figure 3.1.

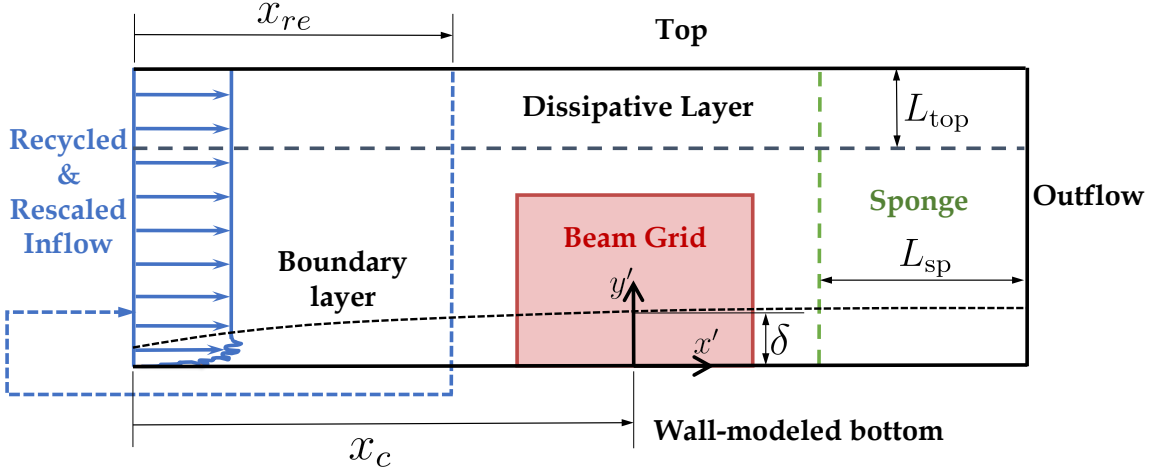


Figure 3.1. Schematic of the computational set-up for turbulent boundary layers.

TABLE 3.1

FLOW CONDITIONS FOR TURBULENT BOUNDARY LAYERS

Flow Condition	1	2	3	4	5
M_∞	0.5	2.0	3.0	4.3	0.5
R_θ	2.7×10^4	6.9×10^4	5.0×10^4	4.3×10^4	3.55×10^3

The reference velocity, density and temperature are the free-stream speed of sound a_∞ , density ρ_∞ and temperature T_∞ , respectively. The reference length scale is taken to be the boundary-layer thickness δ at the optical aperture center $x = x_c$. The domain size is 85δ , 15δ and 3.1δ , in the streamwise, wall-normal and spanwise directions, respectively. The computational domain is sufficiently long in the streamwise direction to accommodate flow structures with relatively large wavelengths.

3.1.1 Boundary Conditions

The “rescale-and-recycle” technique [40, 62] was employed to generate fully-developed turbulent boundary layers within a relatively short domain. A parallelized algorithm for this technique was developed and implemented in CharLES code and it shows high scalability for large numbers of computing cores. This method obtains the flow quantities from a streamwise cut at $x = x_{re}$. The recycling station should be located at a far downstream location from the inlet to allow the development of realistic boundary layer structures. The extracted flowfield at the recycling station is rescaled and reintroduced based on self-similarity of a turbulent boundary layer at the inlet and recycling stations. The rescaling process is controlled by a selected value of the inlet momentum thickness θ_{in} . The self-similarity of the flow quantities in the outer region of the boundary layer is defined in terms of the length scale η , where $\eta = y/\delta$, whereas it is defined in the inner layer in terms of y^+ , where $y^+ = yu_\tau/\nu$. The inlet boundary layer thickness δ_{in} is equal to: $\delta_{in} = \delta_{re} \times \theta_{in}/\theta_{re}$, and the friction velocity at inlet $u_{\tau in}$ is determined as $u_{\tau in} = u_{\tau re} \times (\theta_{re}/\theta_{in})^{0.125}$. The boundary layer thickness δ_{re} , the momentum thickness θ_{re} and the friction velocity $u_{\tau re}$ at the recycling station are calculated from the running-averaged flowfield collected at the recycling station. The value of θ_{in} is adjusted to obtain the target flow conditions in terms of Re_θ at a desirable streamwise location.

Free-stream conditions were imposed on the top boundary. There is a layer of

thickness 5δ , adjacent to the top boundary, where a fully-upwind scheme is used to dissipate acoustic waves. Periodic boundary conditions were imposed in the spanwise direction. The outlet boundary was preceded by a sponge layer [6, 28] to damp out the vortical structures before the flow approaches the outlet. The sponge zone is used to absorb and minimize the acoustic reflections from the outlet boundary. A reference pressure is assigned for the sponge layer and the other reference flow variables are constantly updated using running-averaging. The flow structures in the sponge are damped by adding forcing terms to the right hand sides of the governing equations. These terms are calculated based on the difference between the instantaneous flow quantities and sponge reference values. The sponge thickness is equal to 15δ . Finally, wall-model equations were used to assign approximate adiabatic wall-boundary conditions to the bottom surface. The wall-layer thickness was set to be $y_m = 0.05\delta$.

3.1.2 Spatial and Temporal Resolution

Three levels of grid resolution were employed: a coarse resolution (mesh A), an intermediate mesh (mesh B) and a fine mesh (mesh C), as listed in Table 3.2. In all meshes, the first off-wall spacing is $\Delta y_{min} = 0.01\delta$. The wall-layer thickness, which is equal to $y_m = 0.05\delta$, contains five equally-spaced cells in the wall-normal direction, and thus the matching point is located in a relatively well resolved flowfield and provides more accurate boundary conditions for solving the wall model equations as discussed by Kawai and Larsson [34]. y_{min} and y_m were selected to have the same values for all meshes in order to evaluate the robustness of the calculations for approximate wall boundary conditions. The vertical grid spacing is stretched smoothly from the wall up to $y = 1.5\delta$, followed by a uniform vertical spacing of $\Delta y = \Delta x$ up to $y = 3\delta$, and then a smooth stretching again up to the top boundary. The number of mesh points in y -direction in the range of $0 \leq y \leq 1.5\delta$ is defined as $N_y|_{1.5\delta}$ in Table 3.2. The streamwise spacing, Δx , is constant from the inlet to the

TABLE 3.2

COMPUTATIONAL MESH PARAMETERS FOR VARIOUS LEVELS OF
REFINEMENT FOR BOUNDARY LAYER SIMULATIONS

Mesh	$\Delta x/\delta$	$\Delta z/\delta$	$N_y _{1.5\delta}$	Mesh Size
A	0.077	0.048	45	8.20×10^6
B	0.05	0.032	64	2.18×10^7
C	0.033	0.021	85	6.74×10^7

NOTE: $\Delta y_{min} = \Delta y(0 \leq y \leq 0.05\delta) = 0.01\delta$
 $\Delta y(1.5\delta \leq y \leq 3\delta) = \Delta x$

edge of the sponge, and then it is stretched smoothly towards the outlet inside the sponge layer.

Unity CFL number was applied for time marching in all simulations. The typical value of Δt for the mesh level B is approximately $3.2 \times 10^{-4}\delta/U_\infty$, $9.3 \times 10^{-4}\delta/U_\infty$, $1.08 \times 10^{-3}\delta/U_\infty$ and $1.18 \times 10^{-3}\delta/U_\infty$ for $M_\infty = 0.5, 2.0, 3.0$ and 4.3 , respectively. The time step size decreases by 25% approximately by mesh refinement from level A to level C.

3.1.3 Optical Beam Grid

The aero-optical calculations were performed with an aperture size $20\delta \times 3.1\delta$. The density field was collected every 8 time steps on the beam grid. The beam grid has the exact same mesh resolution as the LES mesh. It covers the whole span, and its height is equal to 4.5δ . The aperture center is located at the midspan and the streamwise location where the target flow conditions are achieved, as depicted in Figure 3.1.

In Section 3.2, a grid convergence and validation study using the three mesh levels (A, B and C) is presented for flow conditions 2 and 5. Section 3.3 mainly discusses the flow and aero-optical predictions of the high-Reynolds-number flow cases listed in Table 3.1 (i.e. flow conditions 1, 2, 3 and 4) utilizing the intermediate mesh (B). Therefore, a total number of nine simulations are considered in this study. The presented statistics of flow and optical quantities were calculated over a time period T_{stats} . The flow and optical parameters of each simulation is listed in Table 3.3.

TABLE 3.3
FLOW AND OPTICAL PARAMETERS FOR BOUNDARY LAYER
SIMULATIONS

Test Case	M_∞	Re_θ	Mesh	θ_{in}/δ	x_{re}/δ	x_c/δ	$T_{\text{stats}}U_\infty/\delta$
Case 1B	0.5	2.7×10^4	B	0.054	15	49.2	483
Case 2A	2.0	6.9×10^4	A	0.038	46	53.1	2108
Case 2B	2.0	6.9×10^4	B	0.038	46	55.4	1394
Case 2C	2.0	6.9×10^4	C	0.038	46	56.9	1725
Case 3B	3.0	5.0×10^4	B	0.031	46	55.4	2171
Case 4B	4.3	4.3×10^4	B	0.023	46	54.6	1779
Case 5A	0.5	3.55×10^3	A	0.035	15	46.0	671
Case 5B	0.5	3.55×10^3	B	0.035	15	48.0	441
Case 5C	0.5	3.55×10^3	C	0.035	15	48.0	447

NOTE: Domain Size: $L_x = 85\delta$, $L_y = 15\delta$ and $L_z = 3.1\delta$

Beam Grid: $L_{x'} = 20\delta$, $L_{y'} = 4.5\delta$ and $L_{z'} = 3.1\delta$

3.2 Mesh Convergence Study

The mesh convergence of the numerical simulations using wall-modeled LES was examined at two different flow conditions: $M_\infty = 0.5$, $Re_\theta = 3.55 \times 10^3$, and $M_\infty = 2.0$, $Re_\theta = 6.9 \times 10^4$. Table 3.4 lists the grid sizes in terms of wall units and time step sizes normalized by outer flow scales. It is obvious the low-Reynolds-number case is adequately resolved for the fine mesh (C) in terms of streamwise and spanwise mesh spacing and time step size, when it is compared to those of wall-resolved LES of Wang and Wang [65] at the same flow conditions.

3.2.1 Grid Sensitivity of Flow Statistics

The time-and-spanwise averaged streamwise component of the velocity field are depicted in Figures 3.2(a) and (b) for the low- and high-Reynolds-number cases, respectively. The mean velocity profiles shows good agreement with the experimental measurements of DeGraaf and Eaton [12] at the closet corresponding value of incompressible Reynolds numbers, $Re_{\theta,inc} = \frac{\mu_\infty}{\langle \mu_w \rangle_z} Re_\theta$ [34], where $\langle \quad \rangle_z$ denotes spanwise averaging, and the logarithmic law, $U^+ = \frac{1}{\kappa} \ln y^+ + C^+$, where $\kappa = 0.41$ and $C^+ = 5.2$. The flow quantities are normalized in terms of wall units: $y^+ = y u_\tau / \langle \overline{\nu_w} \rangle$, and $U^+ = \langle \overline{u} \rangle / u_\tau$. The friction velocity u_τ from the wall model: $u_\tau = \sqrt{\langle \overline{\tau_w} \rangle / \langle \overline{\rho_w} \rangle}$. Wall density ρ_w and wall kinematic viscosity ν_w were calculated on the well-resolved wall-layer mesh. The mean velocity field above the matching point, $y \geq y_m$, was only considered in comparison, because the wall layer is poorly resolved in the LES mesh according to Nyquist criterion as discussed by Kawai and Larsson [34]. Van-Driest transformed velocity, U_{VD}^+ , was calculated to account for compressibility effects [26]. It is defined in the outer layer as:

$$U_{VD}^+ = \int_0^{U^+} \sqrt{\frac{\langle \overline{\rho} \rangle_z}{\langle \overline{\rho_w} \rangle_z}} dU^+. \quad (3.1)$$

TABLE 3.4

GRID SPACINGS AND TIME STEP SIZES IN THE MESH CONVERGENCE
STUDY

	$M_\infty = 0.5, Re_\theta = 3.55 \times 10^3$				$M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$			
	Δx^+	Δy_{min}^+	Δz^+	$\Delta t U_\infty / \delta$	Δx^+	Δy_{min}^+	Δz^+	$\Delta t U_\infty / \delta$
Mesh A	85	11	53	3.35×10^{-3}	693	90	432	1.05×10^{-2}
Mesh B	55	11	36	2.94×10^{-3}	450	90	288	9.30×10^{-3}
Mesh C	36	11	23	2.48×10^{-3}	297	90	189	7.84×10^{-3}

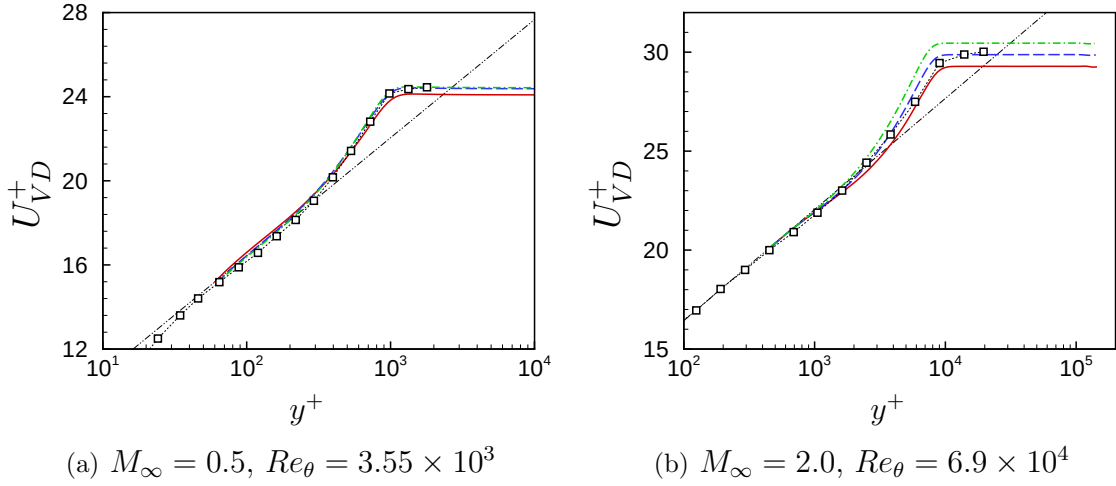


Figure 3.2. Mean streamwise velocity predicted by wall-modeled LES compared to the measurements of incompressible boundary layers of DeGraaff and Eaton [12] at:

(a) $Re_{\theta,inc} = 2.9 \times 10^3$; (b) $Re_{\theta,inc} = 3.1 \times 10^4$. —, mesh A; — —, mesh B; — · —, mesh C. — □ —, experimental measurements. — · —, logarithmic law $U^+ = (1/0.41) \ln y^+ + 5.2$.

For the low-Reynolds-number case, mesh convergence was achieved for the mean velocity profile as demonstrated in Figure 3.2(a), while there was an obvious mesh dependence of mean velocity field in the wake region for the high-Reynolds-number case as shown in Figure 3.2(b), which implies certain flow features in the outer region may be better captured in the fine mesh simulation. More discussion will be presented in this section about the effect of mesh refinement on capturing the large-scale flow structures in the outer region.

The root-mean-square (rms) of streamwise and wall-normal velocity fluctuations of low- and high-Reynolds-number cases obtained from the three meshes are depicted in Figures 3.3(a) and 3.3(b), respectively. They are Van-Driest transformed, i.e.: $u_{rms,VD}^+ = u_{rms}^+ \sqrt{\langle \bar{\rho} \rangle_z / \langle \bar{\rho}_w \rangle_z}$, to consider the compressibility effects. Although the levels of velocity fluctuations show a clear tendency of mesh convergence, it is important to notice that they incrementally rise in the outer region and become closer to the experimental measurements [12].

In Figures 3.4(a) and (b), the rms of density fluctuations of both cases are presented for the three meshes. The results of low-Reynolds number case is compared to the wall-resolved LES results [65]. It is noted that the density fluctuations are close to the predictions from wall-resolved LES, but have a slower decay in the wall-normal direction. The density field shows more sensitivity to grid resolution than the velocity field, especially at the high Reynolds number. While grid convergence has been achieved for velocity statistics on the current meshes as displayed in Figures 3.3(a) and (b), the density still shows a small level of grid dependence.

3.2.2 Grid Sensitivity of Aero-Optical Predictions

In Table 3.5, the normalized $\overline{\text{OPD}}_{rms}$ values are listed for the three levels of mesh resolution. The results show a converging attitude with mesh refinement. The differences in OPD_{rms} between the most refined mesh and the intermediate mesh are

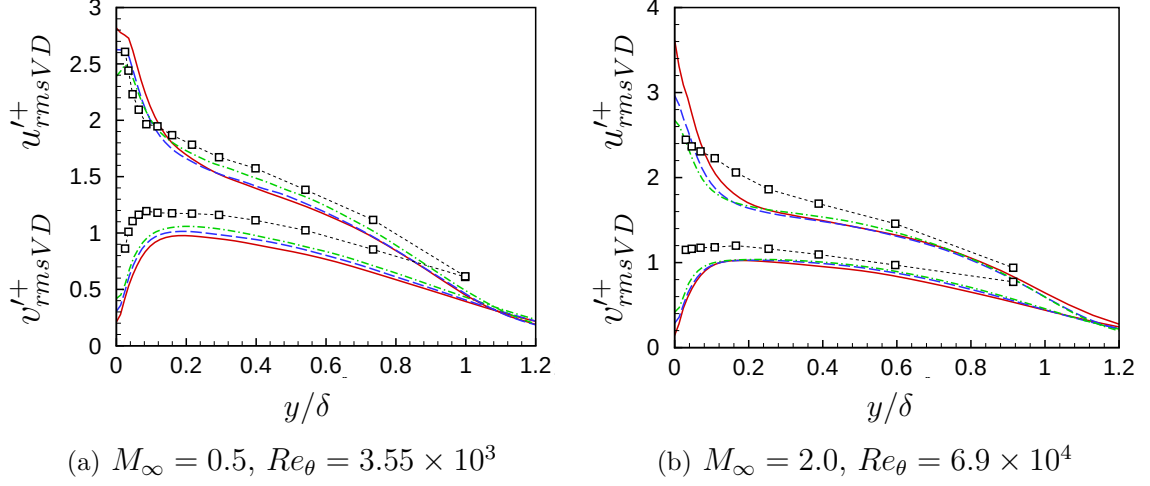


Figure 3.3. Root-mean-square of wall-normal and streamwise velocity fluctuations predicted by wall-modeled LES compared to the measurements of incompressible boundary layers of DeGraaff and Eaton [12] at: (a) $Re_{\theta,inc} = 2.9 \times 10^3$; (b) $Re_{\theta,inc} = 3.1 \times 10^4$. —, mesh A; - - -, mesh B; - · -, mesh C. - - □ - -, experimental measurements.

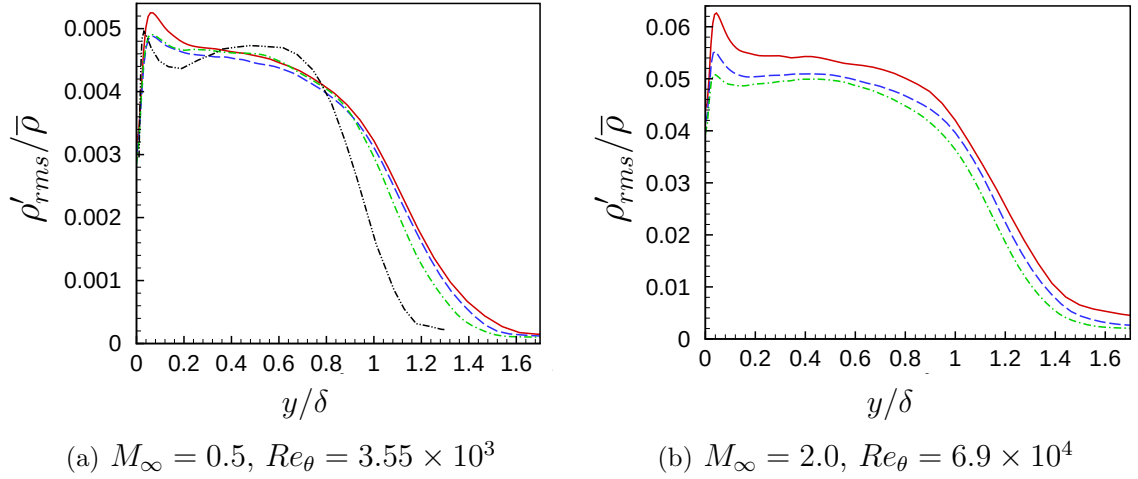


Figure 3.4. Root-mean-square of density fluctuations. Wall-modeled LES: —, mesh A; - - -, mesh B; - · -, mesh C. - · · -, wall-resolved LES of Wang and Wang [65].

0.3% and 6.7% for the low and high Reynolds-number boundary layers, respectively, and thus the intermediate mesh (B) was employed for all the optical calculations of high-Reynolds-number flows as discussed in Section 3.3. The normalized magnitude of optical distortions, $\overline{\text{OPD}}_{rms}/((\rho_\infty/\rho_{SL})\delta M_\infty^2)$, computed by wall-resolved LES at $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$ is 2.72×10^{-6} , which is similar to that of mesh C. The experimental value measured by Gordeyev et al. [22] at $M_\infty = 2.0$ and $Re_\theta = 6.9 \times 10^4$ is 1.42×10^{-6} which is 5% smaller than the predicted value of mesh C.

TABLE 3.5

MAGNITUDE OF OPTICAL DISTORTIONS FOR TURBULENT BOUNDARY
LAYERS AT DIFFERENT LEVELS OF MESH RESOLUTION

Mesh	$\overline{\text{OPD}}_{rms}/((\rho_\infty/\rho_{SL})\delta M_\infty^2)$	
	$M_\infty = 0.5, Re_\theta = 3.55 \times 10^3$	$M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$
A	2.88×10^{-6}	1.86×10^{-6}
B	2.73×10^{-6}	1.59×10^{-6}
C	2.72×10^{-6}	1.49×10^{-6}

The evolution of $\overline{\text{OPD}}_{rms}$ with the wall-normal distance y/δ is illustrated in Figures 3.5(a) and (b) for both flow cases. The grid insensitivity is achieved for all boundary-layer regimes at $M_\infty = 0.5$ and $Re_\theta = 3550$. In Figure 3.5(b), the major discrepancy between meshes B and C is found in the wake region. To demonstrate the relative contributions of different boundary-layer regions to the optical aberrations, $\overline{\text{OPD}}_{rms}$ normalized by its maximum value is plotted as a function of y/δ in Fig-

ures 3.6(a) and (b). It is shown that the logarithmic layer and wake region have the dominant effect in the optical distortions, in agreement with the wall-resolved LES predictions [65]. The three mesh levels show the same profile of $\overline{\text{OPD}}_{rms}$ normalized by its maximum for each flow case.

In order to study the structures of wavefront distortion, the two-point space correlation coefficient of OPD is calculated as:

$$\mathcal{R}_{\text{OPD}}(\Delta x', \Delta z'; x') = \frac{\left\langle \overline{\text{OPD}(x', z', t) \text{OPD}(x' + \Delta x', z' + \Delta z', t)} \right\rangle_z}{\sqrt{\left\langle \overline{\text{OPD}^2(x', z', t)} \right\rangle_z} \sqrt{\left\langle \overline{\text{OPD}^2(x' + \Delta x', z' + \Delta z', t)} \right\rangle_z}}, \quad (3.2)$$

where x' and z' are the streamwise and spanwise locations, respectively, in the local aperture coordinate system.

Figures 3.7(a) and (b) show the streamwise correlation coefficient at the aperture center for both flow cases at the three mesh refinement levels. It is obvious that streamwise correlation length of OPD becomes larger by mesh refinement. However, The correlation of OPD in the spanwise direction at aperture center is insensitive to mesh refinement as shown in Figures 3.8(a) and (b). The relation of optical wavefront structures and density field in boundary layers will be discussed in more details in Section 3.2.3.

A spectral analysis for wavefront distortions was also conducted to demonstrate the mesh dependence of wall-modeled LES predictions. Figures 3.9(a) and (b) depict the frequency power spectral density of OPD at the aperture center at the two flow conditions and three levels of resolution. The frequency power spectral density is determined as:

$$\Phi_{\text{OPD}}(x', f) = \int_{-\infty}^{+\infty} \left\langle \overline{\text{OPD}(x', z', t) \text{OPD}(x', z', t + \Delta t)} \right\rangle_z e^{-i2\pi f \Delta t} d\Delta t. \quad (3.3)$$

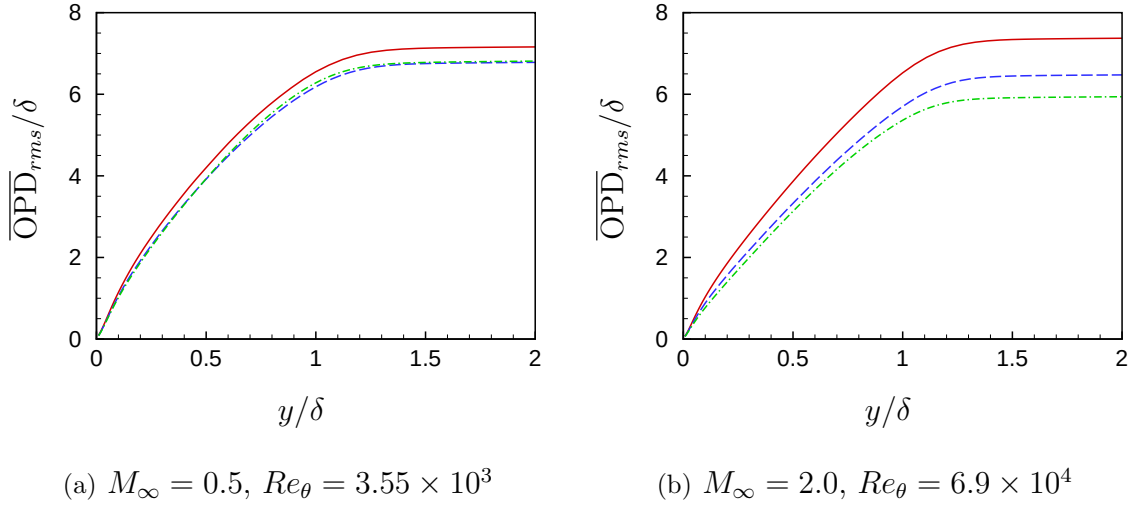


Figure 3.5. Variation of $\overline{\text{OPD}}_{rms}$ across the boundary layer in the wall-normal direction: —, mesh A; - - -, mesh B; - · -, mesh C.

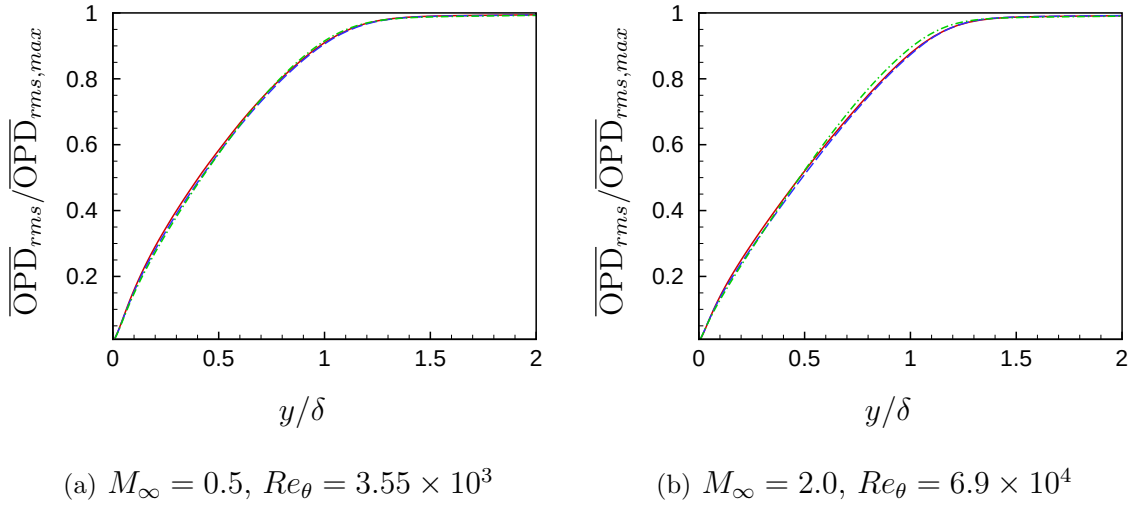
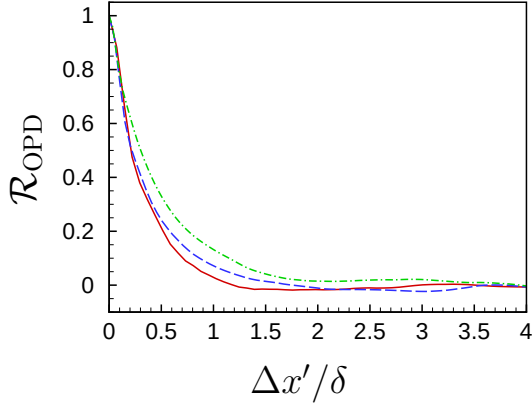
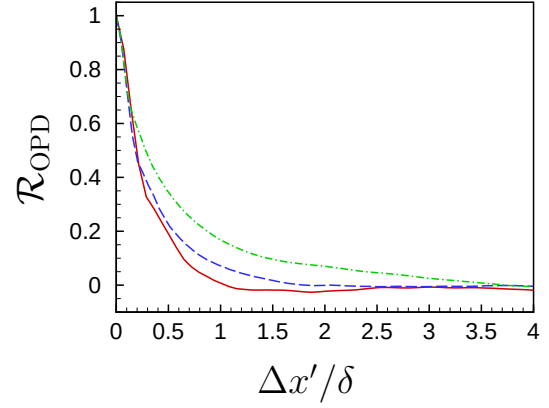


Figure 3.6. Variation of $\overline{\text{OPD}}_{rms}$, normalized by its maximum value, across the boundary layer in the wall-normal direction: —, mesh A; - - -, mesh B; - · -, mesh C.

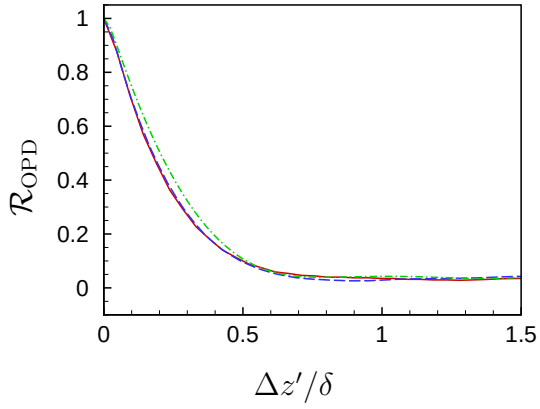


(a) $M_\infty = 0.5$, $Re_\theta = 3.55 \times 10^3$

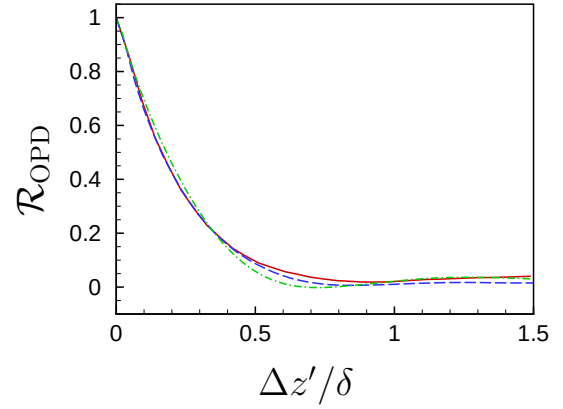


(b) $M_\infty = 2.0$, $Re_\theta = 6.9 \times 10^4$

Figure 3.7. Streamwise space correlation coefficient of OPD at the aperture center:
—, mesh A; - - -, mesh B; - · -, mesh C.



(a) $M_\infty = 0.5$, $Re_\theta = 3.55 \times 10^3$



(b) $M_\infty = 2.0$, $Re_\theta = 6.9 \times 10^4$

Figure 3.8. Spanwise space correlation coefficient of OPD at the aperture center:
—, mesh A; - - -, mesh B; - · -, mesh C.

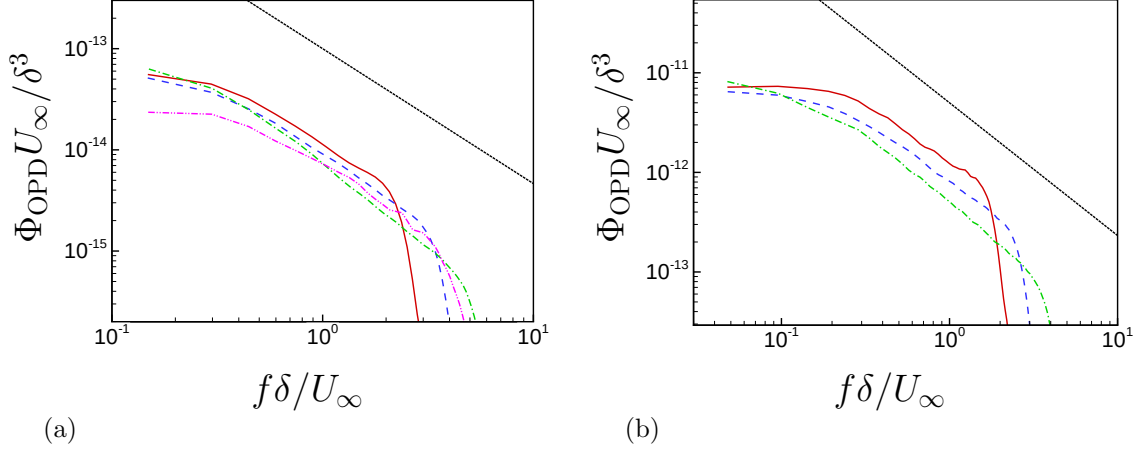


Figure 3.9. Normalized frequency spectra of OPD at $x' = 0$: (a) $M_\infty = 0.5$, $Re_\theta = 3.55 \times 10^3$; (b) $M_\infty = 2.0$, $Re_\theta = 69 \times 10^3$. Predictions of wall-modeled LES: —, mesh A; - - -, mesh B; - · - · -, mesh C. - · · · -, wall-resolved LES [65]; - - - -, line with $-4/3$ slope.

At inertial subrange, the spectra roll-off at $f^{-4/3}$. There is a reasonable agreement at the high-frequency end between the solution of the fine mesh (C) and wall-resolved LES [65] at $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$. It is noted that the spectral level at the low-frequency end from the wall-modeled LES is higher than that from the wall-resolved LES [65], which suggests that the larger streamwise domain in the wall-modeled LES may allow longer streamwise structures to develop and result in the lifted spectral level at the low-frequency end.

3.2.3 Grid Sensitivity of Density Structures

The noticeable elongation of wavefront structures in the streamwise direction by mesh refinement, as shown in Figures 3.7(a) and (b), indicates that the fine mesh captures key flow features that are crucial for the evolution and development of large structures in the outer regions of the boundary layer. This is in accordance with the observed grid sensitivity of the predicted mean and root-mean-square of streamwise velocity in the wake region. To reveal the underlying effect of mesh refinement on the optical wavefront structures, the structures of the density fluctuations are studied in

this section based on the density data collected on the beam grid for different levels of mesh resolution. First, it is required to construct a clear relation between the streamwise correlation of optical distortions and structures of density fluctuations. For homogenous turbulent flowfield, the correlation length of OPD in the streamwise direction can be defined as:

$$\Lambda_{\text{OPD},x'} = \frac{1}{2} \int_{\Delta x'_{\min}}^{\Delta x'_{\max}} \mathcal{R}_{\text{OPD}}(\Delta x', \Delta z' = 0) d\Delta x', \quad (3.4)$$

where $\Delta x'_{\min}$ and $\Delta x'_{\max}$ are the minimum and maximum streamwise separations in the aperture coordinate system. The streamwise coordinates of the local aperture and the LES coordinate systems, for boundary-layer flow case, are related as: $x' = x - x_c$. The streamwise and wall-normal separations are the same for both coordinate systems: $\Delta x = \Delta x'$ and $\Delta y = \Delta y'$. By considering the homogenous turbulence assumption, the streamwise correlation length of OPD is related to the two-point correlation of the density fluctuations in the turbulent boundary layer, as in the following equation:

$$\Lambda_{\text{OPD},x} = \frac{\int_0^{L_{y'}} \langle \overline{\rho'^2(x, y, z, t)} \rangle \int_{-y}^{L_{y'}-y} \int_{\Delta x_{\min}}^{\Delta x_{\max}} \mathcal{R}_{\rho\rho}(\Delta x, \Delta y; y) d\Delta x d\Delta y dy}{2 \left\langle \left(\int_0^{L_{y'}} \rho'(x, y, z, t) dy \right)^2 \right\rangle}, \quad (3.5)$$

where $L_{y'}$ is the beam length. The angle brackets represent spanwise-and-streamwise averaging. The derivation of this relation can be found in Appendix A. Equation(3.5) clarifies the effect of spatial correlation of density field in both the streamwise and beam-propagation directions on the correlation length in streamwise direction. It is valid for large-aperture beams through turbulent flows which are statistically homogeneous in aperture-parallel directions.

At a given location (x, y) , the correlation coefficient of density fluctuations in

the streamwise and beam-propagation directions (or in other words, wall-normal direction, for 90° elevation angle) is calculated based on the collected density field in the beam grid as:

$$\mathcal{R}_{\rho\rho}(\Delta x, \Delta y; x, y) = \frac{\left\langle \overline{\rho'(x, y, z, t) \rho'(x + \Delta x, y + \Delta y, z, t)} \right\rangle_z}{\sqrt{\left\langle \overline{\rho'^2(x, y, z, t)} \right\rangle_z} \sqrt{\left\langle \overline{\rho'^2(x + \Delta x, y + \Delta y, z, t)} \right\rangle_z}}. \quad (3.6)$$

Figures 3.10 and 3.11 illustrate the two-point correlation coefficient of density fluctuations for flow cases: $M_\infty = 0.5, Re_\theta = 3550$ and $M_\infty = 0.5, Re_\theta = 6.9 \times 10^4$, respectively. It is presented as a function of streamwise and vertical separations (Δx and Δy , respectively) for the three levels of mesh refinement (A, B and C) at the streamwise location of aperture center ($x = x_c$) and three wall-normal locations in the outer regions ($y = 0.25\delta, 0.5\delta$ and 0.75δ).

It is shown that mesh refinement captures more anisotropic features of density structures, leads to the increase of the maximum correlation length whose angular position gets more shallow with respect to the streamwise direction. Therefore, a wavefront traveling across these structures will have more streamwisely-correlated structures with higher levels of mesh resolution.

In Figure 3.11(A), (B) and (C), the variation of the density structures with different levels of mesh resolution at the high-Reynolds-number case is more noticeable than those in the low-Reynolds-number case, which is well-resolved, in terms of wall units, in the streamwise and spanwise directions. That indicates that inner flow scales, which are highly-dependent on the Reynolds number and described in terms of wall units, are the key flow features responsible for the existence of the coherent large structures captured in the fine mesh.

For the fine mesh, the two-point correlations of density fluctuations are depicted in Figures 3.10(C) and 3.11(C), for the low and high Reynolds numbers, respectively. The angular orientation of the predicted density structures relative to the

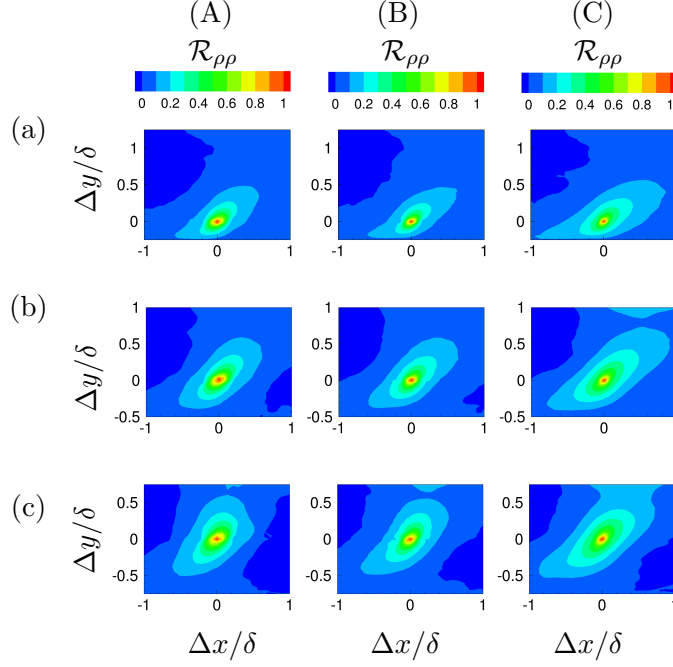


Figure 3.10. Two-point correlation coefficient of density fluctuations at $x = x_c$ and different wall-normal locations at $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$: (a) $y = 0.25\delta$; (b) $y = 0.50\delta$; (c) $y = 0.75\delta$; (A) coarse mesh; (B) intermediate mesh; (C) fine mesh.

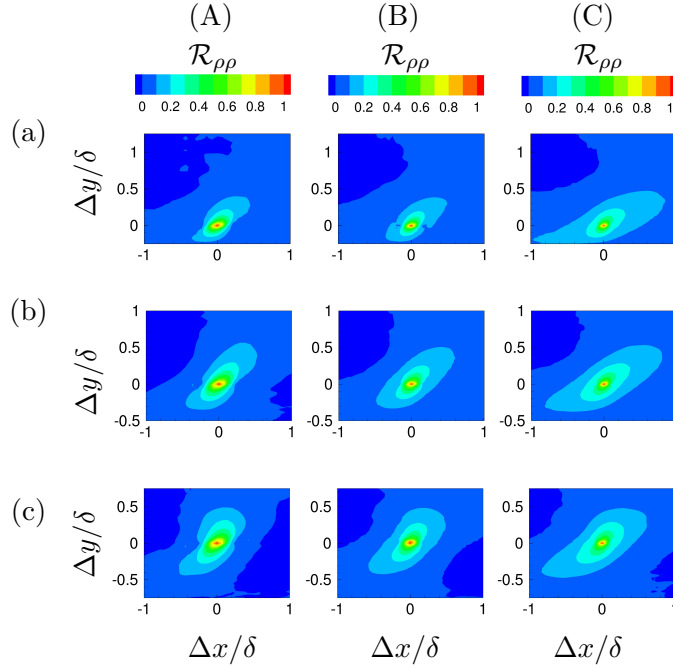


Figure 3.11. Two-point correlation coefficient of density fluctuations at $x = x_c$ and different wall-normal locations at $M_\infty = 2.0$ and $Re_\theta = 6.9 \times 10^4$: (a) $y = 0.25\delta$; (b) $y = 0.50\delta$; (c) $y = 0.75\delta$; (A) coarse mesh; (B) intermediate mesh; (C) fine mesh.

flow direction gets more elevated away from the wall, up to angle 45° , in agreement with the observed orientation of hairpin vortex organization in turbulent boundary layers [2, 14].

Figures 3.12(a) and (b) demonstrate that the wall-normal correlation of density fluctuation is insensitive to mesh refinement. Therefore, the magnitude of optical distortions for a beam traveling at 90° elevation angle varies with mesh refinement only due to grid dependence of the levels of density fluctuations as shown in Figures 3.4(a) and (b), according to Sutton's linking equation [58]:

$$\overline{\text{OPD}}_{rms}^2 = 2 K_{GD}^2 \int_0^{L_{y'}} \rho'^2_{rms}(y) \Lambda_{\rho\rho}(y) dy, \quad (3.7)$$

where

$$\Lambda_{\rho\rho}(y) = \frac{1}{2} \int_{-y}^{L_{y'}-y} \mathcal{R}_{\rho\rho}(\Delta y; y) d\Delta y. \quad (3.8)$$

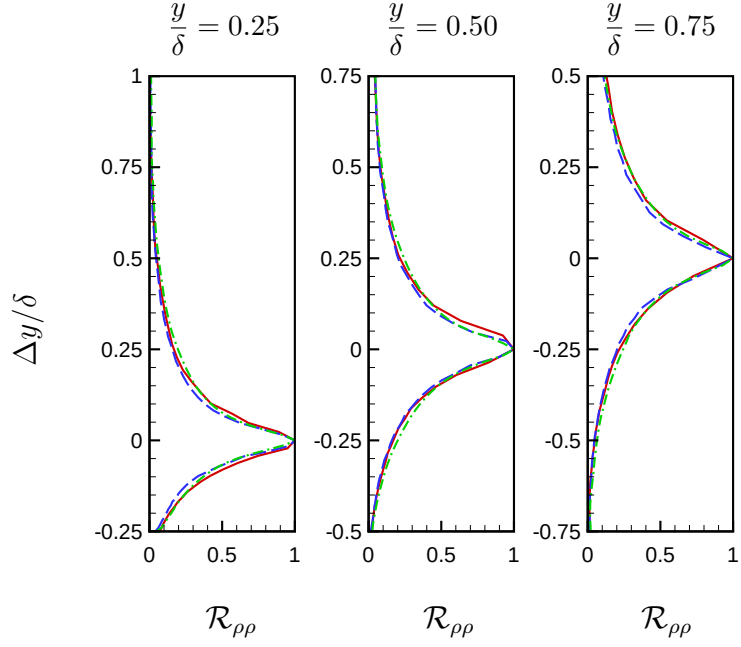
3.3 High-Reynolds-Number Boundary Layers

This section presents and discusses the results from the numerical simulations which employed the intermediate mesh (B). More focus is given to the test cases matching the flow conditions of the experiments conducted by Gordeyev et al. [22, 24, 25].

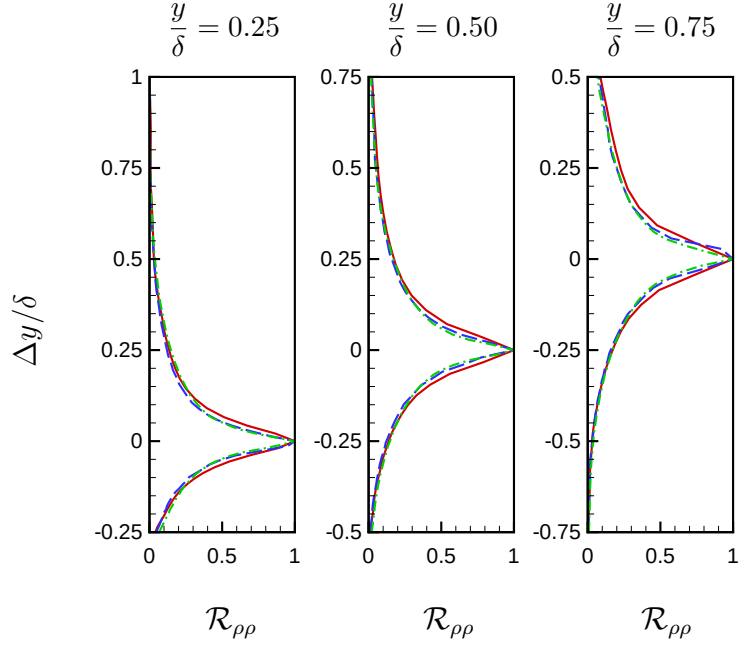
3.3.1 Flow Predictions

In Figure 3.13, the time-and-spanwise averaged streamwise component of the velocity field shows good agreement with the logarithmic law. The outer layer and wake region were accurately predicted.

As demonstrated in Figure 3.14, the evolution of the skin friction coefficient has good matching with Kármán-Schoenherr empirical relation and its modified form based on the high-Reynolds-number experiments reported by Nagib et al. [48]. To



(a) $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$



(b) $M_\infty = 2.0$ and $Re_\theta = 6.9 \times 10^4$

Figure 3.12. Wall-normal space correlation coefficient of density fluctuations at $x = x_c$ and different wall-normal locations: —, mesh A; - - -, mesh B; - · - · -, mesh C;

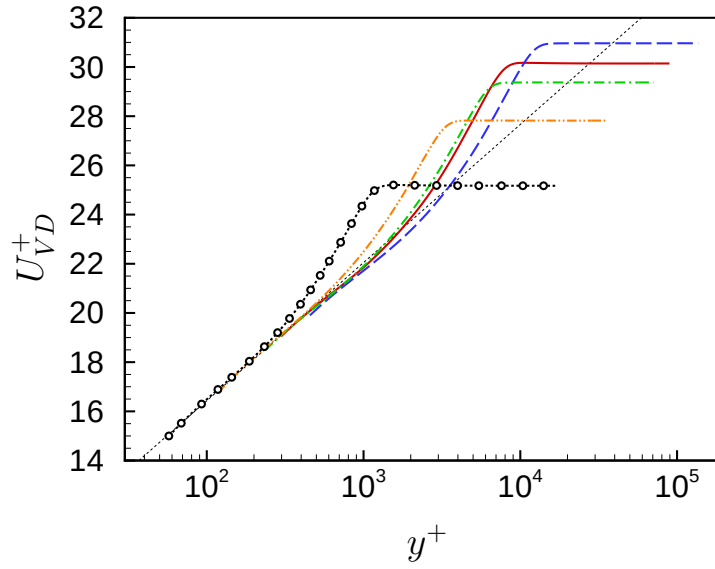


Figure 3.13. Mean streamwise velocity predicted using mesh B: —, $M_\infty = 0.5, Re_\theta = 2.7 \times 10^4$; - - -, $M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$; - · -, $M_\infty = 3.0, Re_\theta = 5.0 \times 10^4$; - · · -, $M_\infty = 4.3, Re_\theta = 4.3 \times 10^4$; - - ○ - -, $M_\infty = 0.5, Re_\theta = 3.55 \times 10^3$. - - - -, Logarithmic law $U^+ = (1/0.41) \ln y^+ + 5.2$.

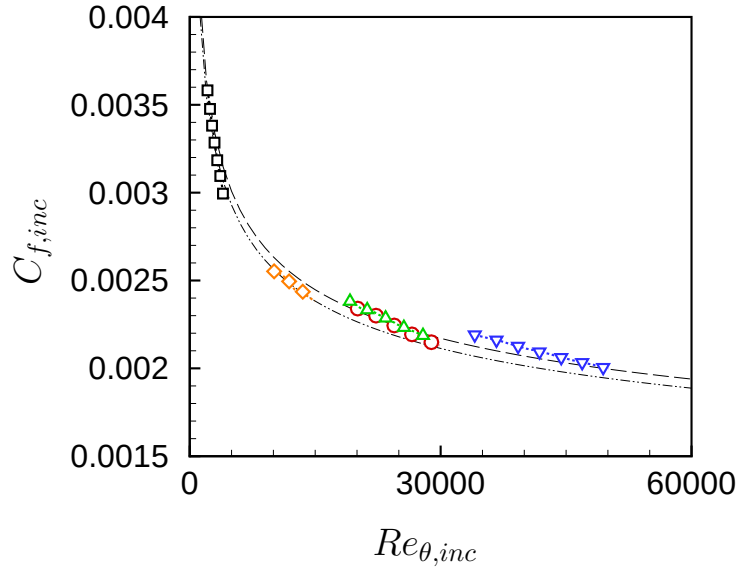


Figure 3.14. Skin friction coefficient predicted using mesh B: - - ○ - -, $M_\infty = 0.5, Re_\theta = 2.7 \times 10^4$; - - ▽ - -, $M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$; - - △ - -, $M_\infty = 3.0, Re_\theta = 5.0 \times 10^4$; - - ◇ - -, $M_\infty = 4.3, Re_\theta = 4.3 \times 10^4$; - - □ - -, $M_\infty = 0.5, Re_\theta = 3.55 \times 10^3$. — — —, Kármán-Schoenherr relation; - · · -, Kármán-Schoenherr modified relation (Nagib et al. [48]).

compare the present compressible flow results with incompressible empirical relation, the van Driest II transformation was applied to transform the compressible skin friction into equivalent incompressible values as [34]:

$$C_{f,inc} = \frac{\langle \overline{T_w} \rangle / T_\infty - 1}{\arcsin^2 \psi} C_f, \quad \psi = \frac{\langle \overline{T_w} \rangle / T_\infty - 1}{\sqrt{\langle \overline{T_w} \rangle / T_\infty (\langle \overline{T_w} \rangle / T_\infty - 1)}} \quad (3.9)$$

In Figure 3.15, the root-mean-square (rms) of the velocity fluctuations at high-Reynolds-numbers are compared to the experimental measurements of DeGraaff and Eaton [12] at the closest value of incompressible Reynolds numbers $Re_{\theta,inc}$. Results from test cases 1B, 2B and 3B are compared to the experimental results at $Re_{\theta,inc} = 3.1 \times 10^4$, and results from test case 4B are compared to those at $Re_{\theta,inc} = 1.3 \times 10^4$. The rms values of streamwise velocity fluctuations (u') are over-predicted in the inner layer, while they are underpredicted in the outer layer. The computed rms values of wall-normal velocity fluctuations (v') are lower than the experimental measurements.

Figure 3.16 displays the root-mean-square of density, temperature and pressure fluctuations normalized by their local mean values. The levels of density fluctuations obviously rise by the increase of the free-stream Mach number, and show consistency with Morkovin's findings for non-hypersonic boundary layers [7]: $\rho' / \bar{\rho} \approx -T' / \bar{T} \approx (\gamma - 1) M_\infty^2 u' / U$. Density and temperature have the same levels of fluctuations inside the boundary layer, while the pressure fluctuations are significantly weaker. These findings are similar to previous observations [65] for low-Reynolds-number subsonic boundary layers using wall-resolved LES. Through the equation of state, density fluctuations, and hence optical distortions, are mainly affected by the temperature fluctuations inside the boundary layer, and then becomes more affected by those of the pressure outside it. It is also shown that a secondary peak of the density and temperature fluctuations in the outer layer rises as the Mach number increases.

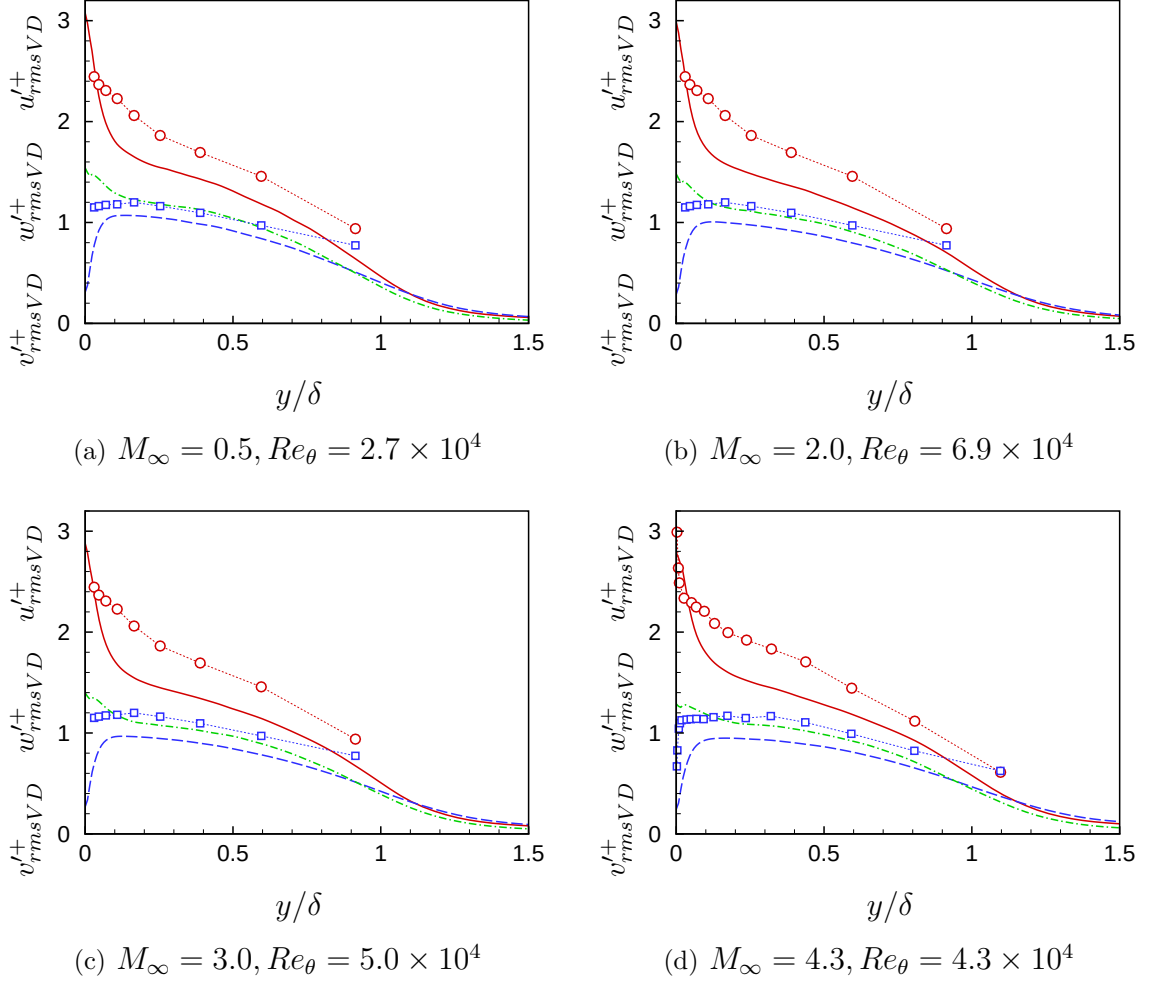


Figure 3.15. Root-mean-square of wall-normal, spanwise and streamwise velocity fluctuations on mesh B: —, u'_{rmsVD} ; — —, v'_{rmsVD} ; — · —, w'_{rmsVD} . Experimental measurements of Degraaff and Eaton [12] at $Re_{\theta,inc} = 3.1 \times 10^4$ (a,b and c) and $Re_{\theta,inc} = 1.3 \times 10^4$ (d): - - ○ - -, u'_{rms} ; - - □ - -, v'_{rms} .

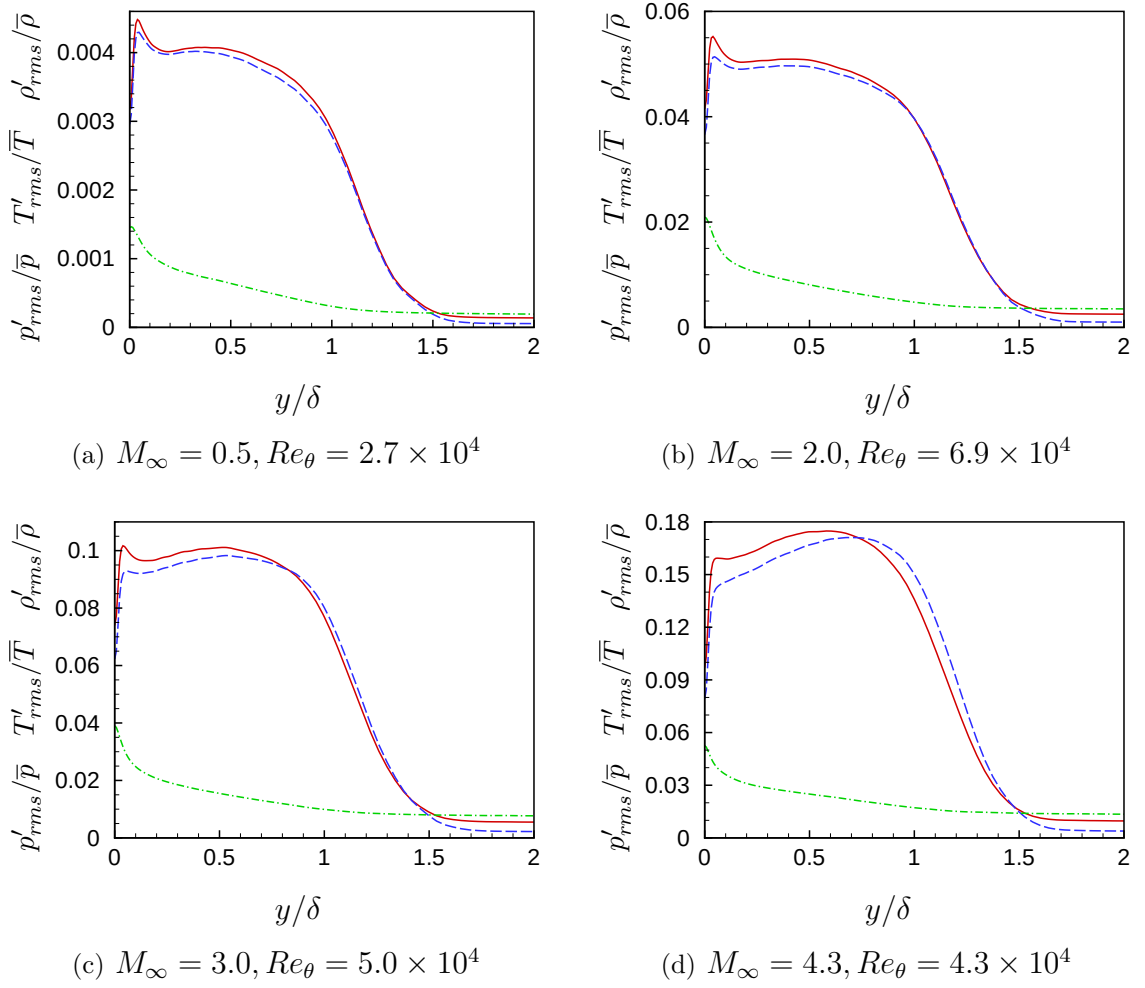


Figure 3.16. Root-mean-square of pressure, temperature and density fluctuations on mesh B: —, $\rho'_{rms}/\bar{\rho}$; - - -, $T'_{rms}/\bar{T}$; - · -, $p'_{rms}/\bar{p}$.

3.3.2 Aero-Optical Predictions

Figure 3.17 depicts the predicted $\overline{\text{OPD}}_{rms}$ of subsonic and supersonic boundary layers. Compared to the corresponding experimental measurements, a reasonable agreement is observed at lower Mach numbers, but the rate of decrease of the predicted optical distortions with Mach number is slower than that of the experimental data. The numerical results at high- Re cases at $M_\infty = 0.5$ and 2.0 (cases 1B and 2B) fall within the margin of accuracy of experimental estimations. However, they were overpredicted at $M_\infty = 3.0$ and 4.3 (cases 3B and 4B) by 28% and 40%, respectively. Additional work is needed to fully address the discrepancy between the experimental estimations [25] and numerical predictions of $\overline{\text{OPD}}_{rms}/\delta$. Nevertheless, one potential reason is the necessity of development of improved well-posed approximate adiabatic wall boundary conditions accounting of thermal dissipation in the near-wall region. Therefore, it is required to have supplementary reference test cases of high-Mach-numbers non-hypersonic turbulent boundary layers computed by wall-resolved LES or DNS to fully evaluate wall-modeled LES predictions for the levels of density fluctuations in this flow regime.

In Figure 3.18, The two-point space correlation of OPD is displayed to study the wavefront distortion at different Mach numbers and Reynolds numbers. The two-point space correlation of OPD at the aperture center, $x' = 0$, is calculated according to Eq.(3.2). As the Mach number gets higher, the wavefront structures have slightly larger correlation lengths in streamwise and spanwise directions.

As it is acceptable to assume that wavefront distortions are homogenous for turbulent boundary layers [24, 65]. Figure 3.19 compares between streamwise-and-spanwise averaged two-point correlation of OPD predicted by the numerical simulation to the results of the subsonic boundary layer experiments conducted by Gordeyev et al. [24] at the transonic in-draft wind-tunnel facility at the Hestert Laboratory for Aerospace Research at University of Notre Dame. The calculations for the spatial correlation is

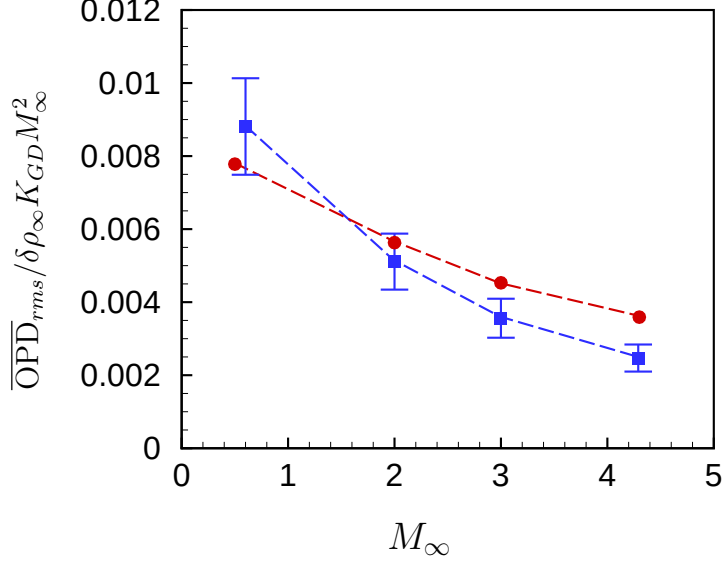


Figure 3.17. Normalized $\overline{\text{OPD}}_{rms}$ for high- Re turbulent boundary layers: -- ● --, wall-modeled LES; -- ■ -- with error bars, experimental measurements [22, 25].

carried out for square aperture of smaller area compared to the full numerical aperture. The streamwise-averaged two-point space correlation of OPD is computed as follows:

$$\mathcal{R}_{\text{OPD}}(\Delta x', \Delta z') = \frac{\langle \overline{\text{OPD}(x', z', t) \text{OPD}(x' + \Delta x', z' + \Delta z', t)} \rangle}{\sqrt{\langle \overline{\text{OPD}^2(x', z', t)} \rangle} \sqrt{\langle \overline{\text{OPD}^2(x' + \Delta x', z' + \Delta z', t)} \rangle}}, \quad (3.10)$$

where the angle brackets represent spanwise and streamwise averaging. The numerical results shows lower streamwise correlation for small values of spatial separations, and show no obvious negative correlation for larger separations compared to the experimental results. The spanwise correlations of optical wavefront are significantly less than those of the experiments. One potential reason for this discrepancy is that the computational domain is set to be periodic across a span of total length 3.1δ , where δ is the boundary layer thickness at aperture center, while the experimental data were taken with wind-tunnel walls with a distance of more than 25δ [24].

Figure 3.20 presents the iso-contours of the space-time correlation of OPD at the

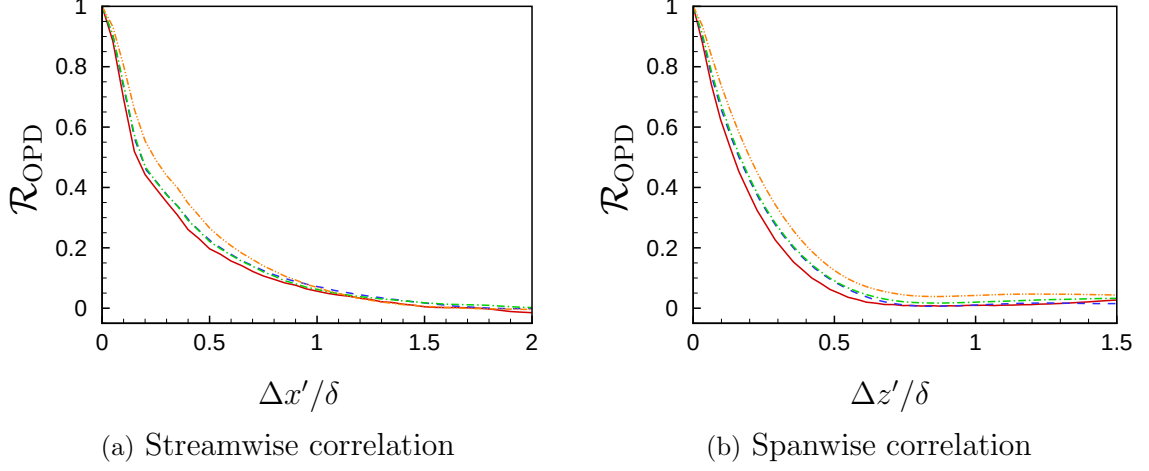


Figure 3.18. Correlation coefficient of OPD at the aperture center for aperture size $10\delta \times 3.1\delta$ on mesh B: —, $M_\infty = 0.5, Re_\theta = 2.7 \times 10^4$; - - -, $M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$; - · - · -, $M_\infty = 3.0, Re_\theta = 5.0 \times 10^4$; · · · · ·, $M_\infty = 4.3, Re_\theta = 4.3 \times 10^4$.

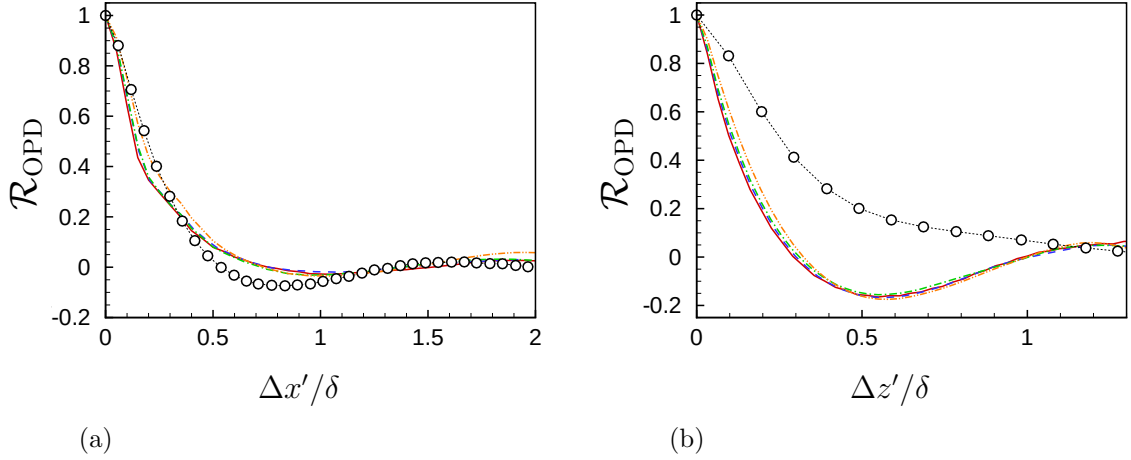


Figure 3.19. Correlation coefficient of OPD: (a) streamwise space correlation with an aperture $2.0\delta \times 2.0\delta$; (b) spanwise space correlation with an aperture $1.4\delta \times 1.4\delta$. —, $M_\infty = 0.5, Re_\theta = 2.7 \times 10^4$; - - -, $M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$; - · - · -, $M_\infty = 3.0, Re_\theta = 5.0 \times 10^4$; · · · · ·, $M_\infty = 4.3, Re_\theta = 4.3 \times 10^4$. - - o - -, experimental measurements of Gordeyev et al. [24] for subsonic boundary layers.

streamwise location of aperture center ($x' = 0$) for two high-Reynolds-number test case at $M_\infty = 0.5$ and 4.3 (cases 1B and 4B). The space-time correlation of OPD is calculated as:

$$\mathcal{R}_{\text{OPD}}(\Delta x', \Delta t; x') = \frac{\left\langle \overline{\text{OPD}(x', z', t) \text{OPD}(x' + \Delta x', z', t + \Delta t)} \right\rangle_z}{\sqrt{\left\langle \overline{\text{OPD}^2(x', z', t)} \right\rangle_z} \sqrt{\left\langle \overline{\text{OPD}^2(x' + \Delta x', z', t + \Delta t)} \right\rangle_z}}. \quad (3.11)$$

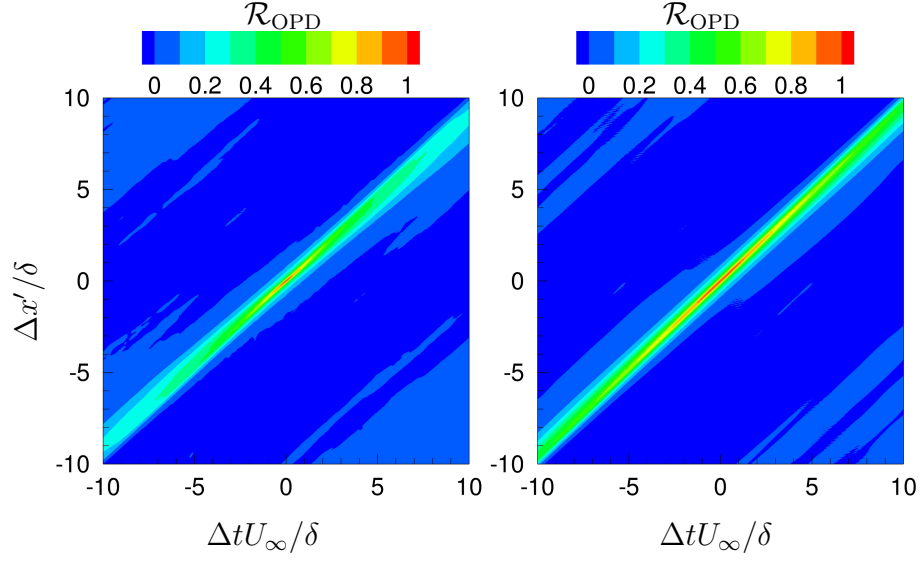
Based on the slope of the peak branches of the space-time correlation, the convective speed of the optical structures U_c can be determined as a function of streamwise spatial separation as follows [65]:

$$U_c(\Delta x') = \frac{\Delta x'}{\Delta t_c}, \quad \left. \frac{\partial \mathcal{R}_{\text{OPD}}(\Delta x', \Delta t)}{\partial \Delta t} \right|_{\Delta t = \Delta t_c} = 0, \quad \left. \frac{\partial^2 \mathcal{R}_{\text{OPD}}(\Delta x', \Delta t)}{\partial \Delta t^2} \right|_{\Delta t = \Delta t_c} < 0. \quad (3.12)$$

And, it is calculated as a function of temporal separation as:

$$U_c(\Delta t) = \frac{\Delta x'_c}{\Delta t}, \quad \left. \frac{\partial \mathcal{R}_{\text{OPD}}(\Delta x', \Delta t)}{\partial \Delta x'} \right|_{\Delta x' = \Delta x'_c} = 0, \quad \left. \frac{\partial^2 \mathcal{R}_{\text{OPD}}(\Delta x', \Delta t)}{\partial \Delta x'^2} \right|_{\Delta x' = \Delta x'_c} < 0. \quad (3.13)$$

U_c increases with increasing temporal and spatial separations because the optical distortions are mainly affected by the relatively large flow structures. Its values, normalized by the free-stream velocity, slightly increases as the Mach number increases in agreement with the experimental observations [22, 25]. The experimental measurements of U_c/U_∞ are 0.83, 0.89 and 0.91 at $M_\infty = 0.5, 3.0$ and 4.3, respectively, using 2-D Shack-Hartmann waveFront sensors [24, 25] and 0.84 at $M_\infty = 2.0$ using Malley probes for a separation of 0.65δ [22, 25]. The numerical estimations of the averaged U_c/U_∞ using the intermediate mesh B are 0.93, 0.94, 0.95 and 0.96 for $M_\infty = 0.5, 2.0, 3.0$ and 4.3, respectively. That shows U_c/U_∞ is overpredicted with maximum difference of 12% compared to experimental measurements.



(a) $M_\infty = 0.5, Re_\theta = 2.7 \times 10^4$ (b) $M_\infty = 4.3, Re_\theta = 4.3 \times 10^4$

Figure 3.20. Space-time correlation of OPD at the aperture center on mesh B.

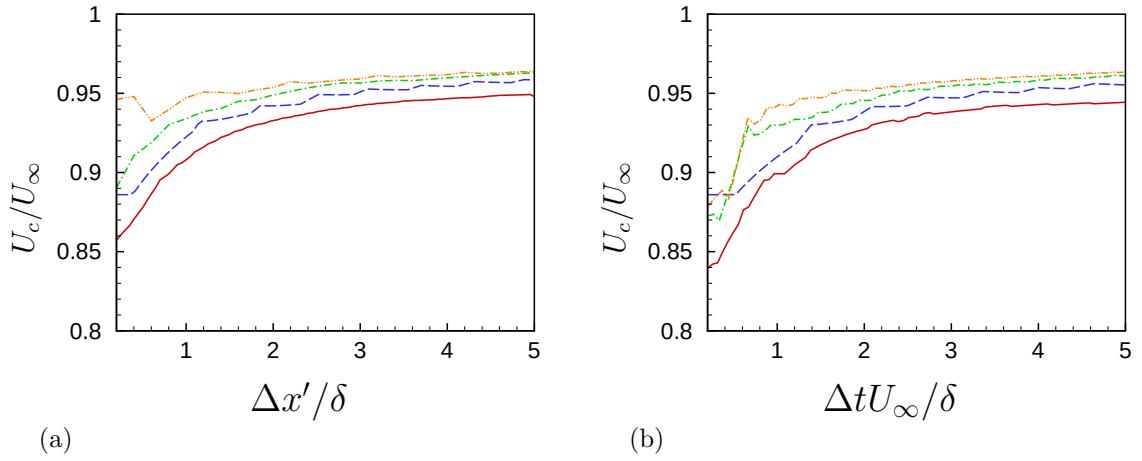


Figure 3.21. Convective velocities of OPD as a function of: (a) streamwise separation; and (b) temporal separation, on mesh B. —, $M_\infty = 0.5, Re_\theta = 2.7 \times 10^4$; - - -, $M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$; - . - , $M_\infty = 3.0, Re_\theta = 5.0 \times 10^4$; . . . , $M_\infty = 4.3, Re_\theta = 4.3 \times 10^4$.

3.4 Flow Structures in Turbulent Boundary Layers

Given that the grid sensitivity analysis indicates that wall-modeled LES provides reasonably good mesh-convergent flow and aero-optical predictions. The study is extended to analyze the density and velocity structures in boundary layers employing the most refined mesh at the two flow conditions considered in study of the mesh refinement effect.

The streamwise velocity field is collected in the beam grid for flow cases: $M_\infty = 0.5, Re_\theta = 3550$ and $M_\infty = 0.5, Re_\theta = 6.9 \times 10^4$ for the fine mesh C. The correlation coefficient of streamwise velocity fluctuations is calculated as:

$$\mathcal{R}_{uu}(\Delta x, \Delta y; x) = \frac{\left\langle \overline{u'(x, y, z, t)u'(x + \Delta x, y + \Delta y, z, t)} \right\rangle_z}{\sqrt{\left\langle \overline{u'^2(x, y, z, t)} \right\rangle_z} \sqrt{\left\langle \overline{u'^2(x + \Delta x, y + \Delta y, z, t)} \right\rangle_z}}. \quad (3.14)$$

Iso-contours of two-point correlation of streamwise velocity fluctuations compared to those of density fluctuations as calculated by Eq.(3.6) are illustrated for the fine mesh in Figures 3.22 and 3.23 at the flow conditions: $M_\infty = 0.5, Re_\theta = 3.55 \times 10^3$ and $M_\infty = 2.0, Re_\theta = 6.9 \times 10^4$, respectively. As previously discussed, the predicted density structures displayed in the figures are consistent with the orientation of boundary-layer large-scale motions (LSM), such as hairpin vortices [3, 37]. However, the correlation of streamwise velocity fluctuations are significantly elongated in the streamwise direction. That implies that very-long flow features becomes remarkable only in the velocity field statistics.

These elongated structures observed in the velocity field have some similar features to that of the experimental observations [2, 14, 27] and DNS results [39, 57] of the flow regime underneath the hairpin vortex organization and characterized by of negative streamwise velocity fluctuations induced by its vortical structures.

The characteristic length scale of these flow structures at different wall-normal

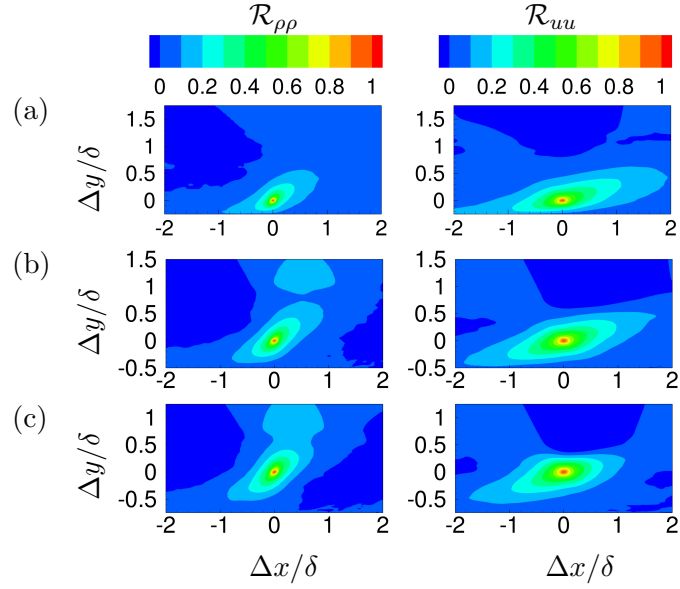


Figure 3.22. Two-point correlation coefficient of density and streamwise velocity fluctuations on mesh C at $x = x_c$ and different wall-normal locations at $M_\infty = 0.5$, $Re_\theta = 3.55 \times 10^3$: (a) $y = 0.25\delta$; (b) $y = 0.50\delta$; (c) $y = 0.75\delta$.

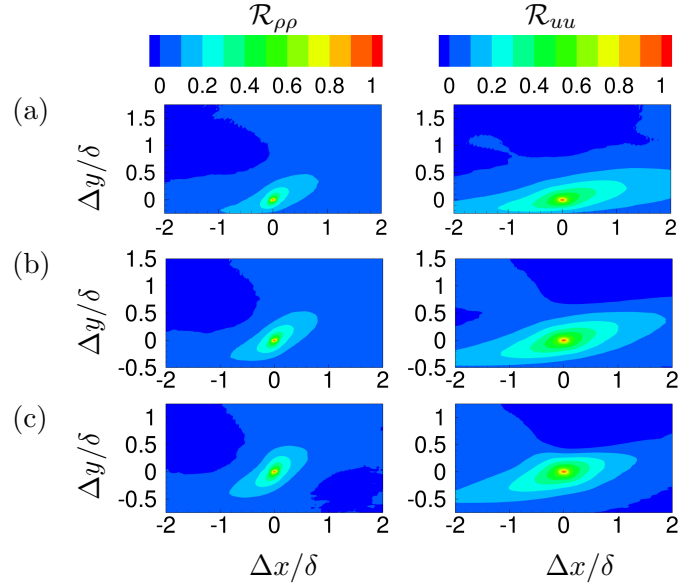


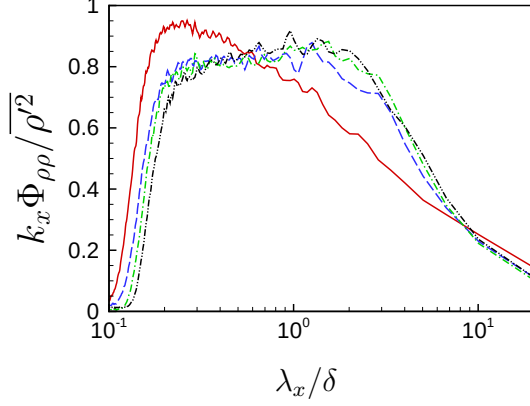
Figure 3.23. Two-point correlation coefficient of density and streamwise velocity fluctuations on mesh C at $x = x_c$ and different wall-normal locations at $M_\infty = 2.0$, $Re_\theta = 6.9 \times 10^4$: (a) $y = 0.25\delta$; (b) $y = 0.50\delta$; (c) $y = 0.75\delta$.

locations can be determined by the peaks of the pre-multiplied power spectra of density and streamwise velocity fluctuations. The wavenumber power spectrum of density and streamwise fluctuations, at given streamwise and wall-normal locations are defined as $\Phi_{\rho\rho}$ and Φ_{uu} , respectively:

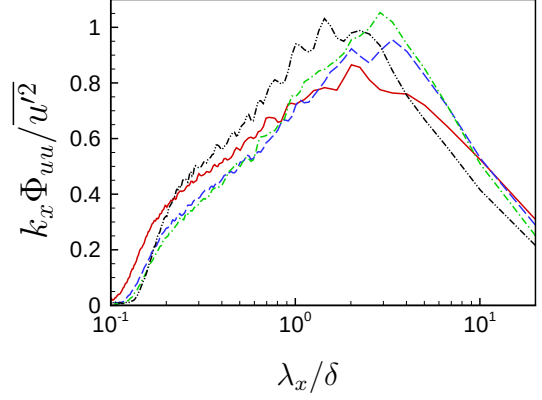
$$\Phi_{\rho\rho}(x, y, k_x) = \int_{-\infty}^{+\infty} \left\langle \overline{\rho'(x, y, z, t) \rho'(x + \Delta x, y, z, t)} \right\rangle_z e^{(-ik_x \Delta x)} d\Delta x, \quad (3.15)$$

$$\Phi_{uu}(x, y, k_x) = \int_{-\infty}^{+\infty} \left\langle \overline{u'(x, y, z, t) u'(x + \Delta x, y, z, t)} \right\rangle_z e^{(-ik_x \Delta x)} d\Delta x. \quad (3.16)$$

At $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$, where the flowfield is adequately resolved in the logarithmic and wake regions, the pre-multiplied spectra of density and streamwise velocity fluctuations are normalized by their local mean-squares, and presented in Figures 3.24(a) and (b), respectively, at the aperture center ($x = x_c$) and selected off-wall locations in the outer region. The peaks of the pre-multiplied density spectra in the lower wake region, at $y = 0.5\delta$ and $y = 0.75\delta$, correspond to wavelengths $\lambda_x = \frac{2\pi}{k_x} \sim \mathcal{O}(1\delta - 2\delta)$. On the other side, the peaks of the pre-multiplied streamwise velocity spectra at $y = 0.5\delta$ and 0.75δ are located at wavelengths $\lambda_x \sim \mathcal{O}(3\delta - 4\delta)$. Figures 3.25(a) and (b) depict the corresponding pre-multiplied spectra at $M_\infty = 2.0$ and $Re_\theta = 6.9 \times 10^4$ for density and streamwise velocity fields, respectively. The peaks of density pre-multiplied spectra are found at wavelengths $\lambda_x \sim \mathcal{O}(0.2\delta - 0.3\delta)$. The peaks of streamwise velocity pre-multiplied spectra in the wake region are corresponding to wavelengths $\lambda_x \sim \mathcal{O}(3\delta - 4\delta)$. It is shown there are differences in the pre-multiplied spectra between the low- and high-Reynolds-number cases. One potential reason of that is the flowfield at the high Reynolds number is not sufficiently resolved using the most refined mesh in order to fully capture the energy content of different flow structures. This suggestion is supported by the fact that the spatial resolution of mesh C in terms of wall units is relatively poor compared to that of adequately resolved LES in the high-Reynolds-number case, as shown in Table 3.4.

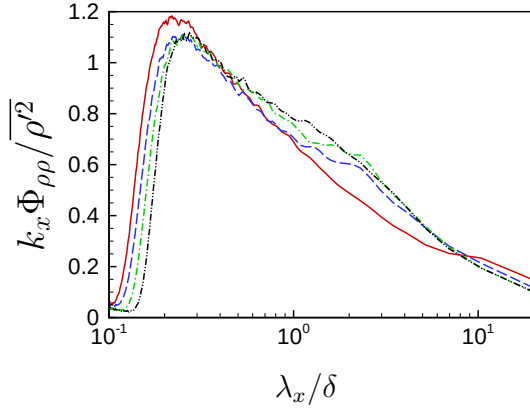


(a) Density spectra

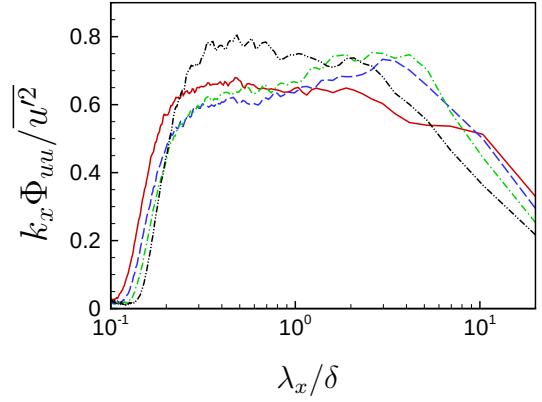


(b) Streamwise velocity spectra

Figure 3.24. Pre-multiplied streamwise wavenumber spectra predicted for the case of $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$ on mesh C at $x = x_c$ and different wall-normal locations: —, $y/\delta = 0.25$; - - -, $y/\delta = 0.50$; - · - · -, $y/\delta = 0.75$; · · · ·, $y/\delta = 1.00$.



(a) Density spectra



(b) Streamwise velocity spectra

Figure 3.25. Pre-multiplied streamwise wavenumber spectra predicted for the case of $M_\infty = 2.0$ and $Re_\theta = 6.9 \times 10^4$ on mesh C at $x = x_c$ and different wall-normal locations: —, $y/\delta = 0.25$; - - -, $y/\delta = 0.50$; - · - · -, $y/\delta = 0.75$; · · · ·, $y/\delta = 1.00$.

For a broader view, Figures 3.26 and 3.27 illustrate the iso-contours of the normalized pre-multiplied spectra of density and streamwise fields to demonstrate the variation of characteristic length scale with off-wall distance for both flow cases, instead of selecting certain y -locations. For the low-Reynolds-number case, the characteristic length scale of density structures does not exceed 3δ in the outer region, whereas it is on the order of 0.3δ for the high-Reynolds-number case. On the other hand, the characteristic length of the streamwise velocity field is $\lambda_x \sim \mathcal{O}(3\delta - 5\delta)$ for both flow cases.

Balakumar and Adrian [3] defined structures of lengths ranging from approximately $\lambda_x = 1\delta$ up to 3δ as large-scale motions (LSM). Also they defined structures greater than 3δ as very-large-scale motions (VSLM).

Although The predicted structures of density fluctuations captures the characteristic features of large-scale flow structures, no key features of very-large-scale motions are captured in the density field. On the other side, the very-large-scale structures produced are recognized in the pre-multiplied spectra of streamwise velocity fluctuations whose characteristic maximum wavelength is of order $3\delta - 5\delta$.

This observation can probably be explained by that density fluctuation are mainly dominated by temperature fluctuations inside the boundary layer, and the turbulent temperature field is mainly connected to the dissipation of vortical structures. Given that the vorticity is almost negligible in VLSM's compared to those of LSM's such as hairpin vortices, the temperature and hence density statistics are dominated by the characteristic features of large-scale flow structures.

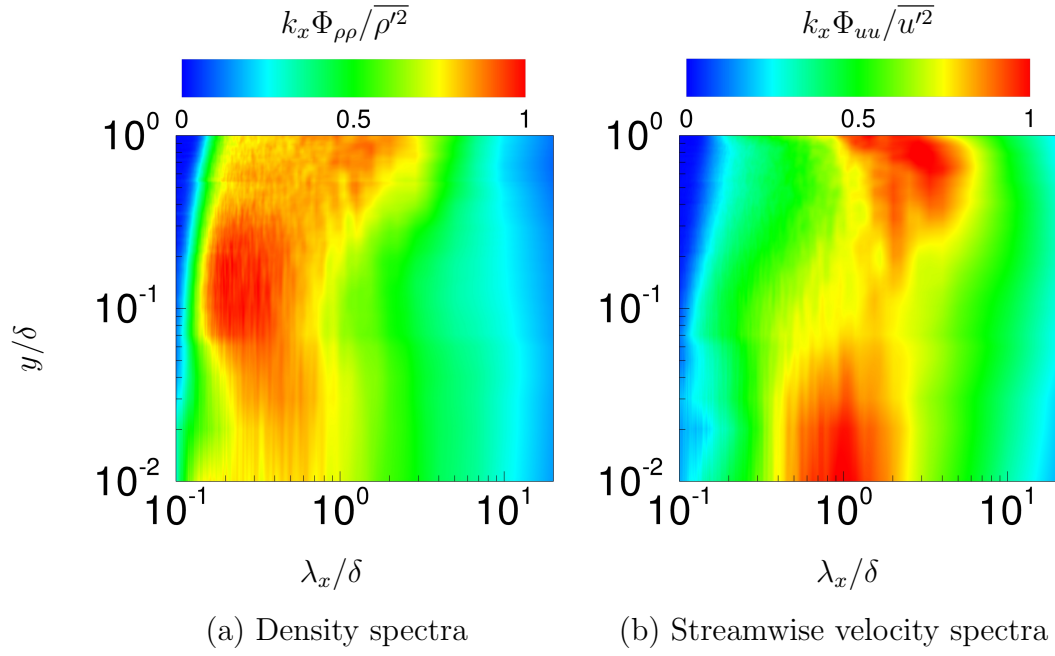


Figure 3.26. Contours of pre-multiplied streamwise wavenumber spectra at $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$ on mesh C at $x = x_c$ along the wall-normal direction.

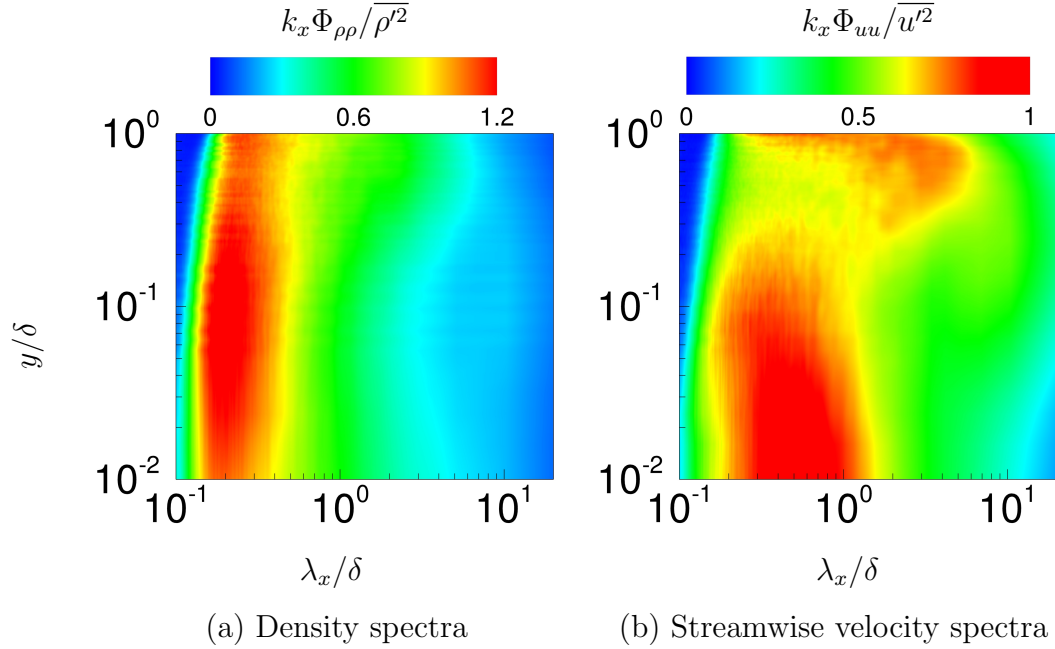


Figure 3.27. Contours of pre-multiplied streamwise wavenumber spectra at $M_\infty = 2.0$ and $Re_\theta = 6.9 \times 10^4$ on mesh C at $x = x_c$ along the wall-normal direction.

3.5 Conclusions

Wall-modeled LES is employed to compute subsonic and supersonic turbulent boundary layers at Mach numbers $M_\infty = 0.5, 2.0, 3.0$ and 4.3 and Reynolds numbers ranging from $Re_\theta = 3.55 \times 10^3$ to 6.9×10^4 . A mesh convergence study is conducted at two flow conditions: the first one is at $M_\infty = 0.5$ and $Re_\theta = 3.55 \times 10^3$, and the second one is at $M_\infty = 2.0$ and $Re_\theta = 6.9 \times 10^4$. The flowfield and aero-optical predictions converge faster in the low-Reynolds-number case. The grid sensitivity study, using three level of mesh resolution, illustrates the tendency of flow and aero-optics predictions to converge with mesh refinement. Although, it is found that predictions of density fluctuations and optical distortions are more demanding in terms of mesh resolution compared to velocity fluctuations, reasonably accurate results can be obtained using the intermediate mesh resolution for the turbulent density field and associated aero-optics. The current numerical results are in reasonable agreement with previous results from experimental measurement [12, 22, 24, 25] at practical values of Reynolds numbers and wall-resolved LES at a low Reynolds number [65]. The signature of large flow structures in the outer layer is observed in the flow-field and optics statistics. The statistics of the density field captures the characteristic features of the large-scale motions (LSM) whose streamwise wavelength is less than 3δ . However, the very-large-scale motions (VLSM) are only recognizable in the streamwise velocity showing a characteristic wavelength greater than 3δ and ranging from 4δ to 5δ . A potential cause of that is the temperature fluctuations, and hence density fluctuations, are mainly related to the dissipation of large-scale vortical structures inside turbulent boundary layers, and there are no significant temperature and density fluctuations associated with the very-long structures.

CHAPTER 4

SUBSONIC FLOWS OVER CYLINDRICAL A TURRET

Wall-modeled LES was conducted for the subsonic flow over a partially protruding cylindrical turret which was experimentally studied by Gordeyev et al. [19, 21]. A cylindrical turret of radius $R = 2$ inches was mounted between two flat plates of different altitudes. As shown in Figure 4.1, a flat window with a length of R was installed on the cylinder, which could be rotated to project the laser beam at different elevation angles, θ , ranging from 90° to 150° . The incoming flow separates at the sharp edge forming large coherent structures due to Kelvin-Helmholtz instability [15], leading to optical distortion for a laser beam traveling through the separated flow region. The velocity statistics profiles adjacent to the optical window and optical distortions were experimentally measured at different elevation angles over a Mach number range of 0.4 to 0.7, and Reynolds number range of 4.4×10^5 to 8.1×10^5 . This two-dimensional configuration provides a canonical test case to validate aero-optical predictions for subsonic separated flows. In the present study, this flow was computed employing wall-modeled LES at Mach number $M_\infty = 0.5$ and an experimental Reynolds number. Mesh convergence study was conducted in order to determine the required spatial resolution for flow and aero-optical predictions. This work was partially presented in [33].

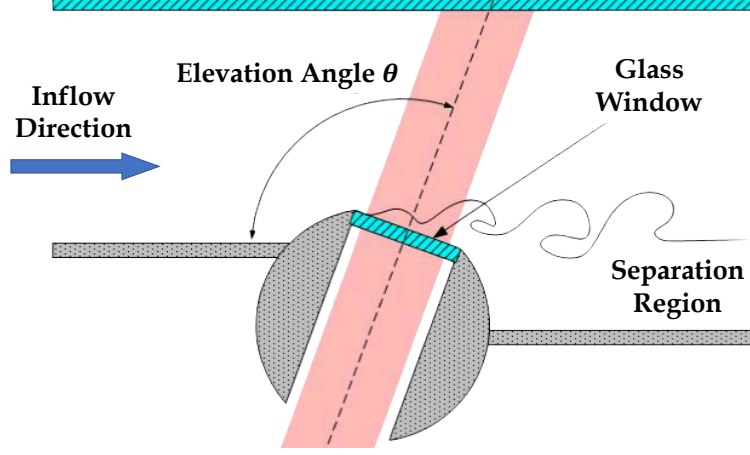


Figure 4.1. Schematic of subsonic flow over a cylindrical turret with a flat window [21]

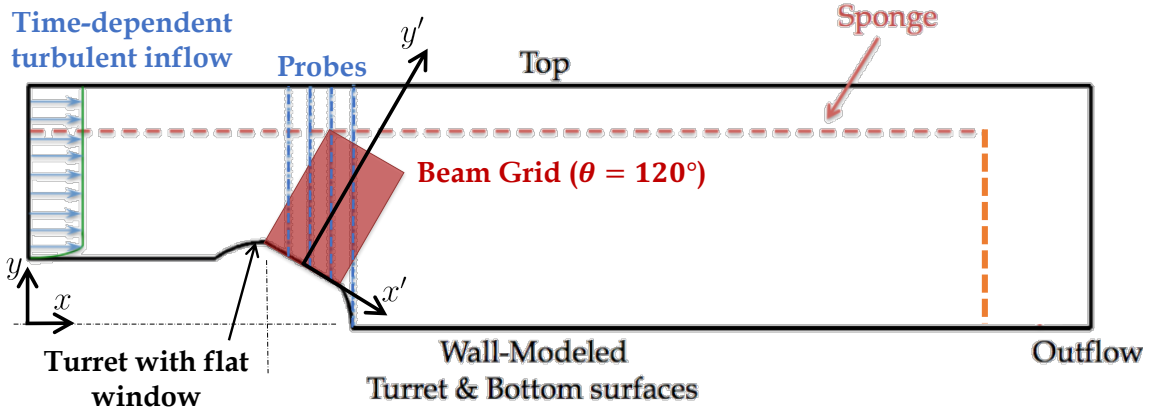


Figure 4.2. Schematic of the computational setup for subsonic flow over a cylindrical turret with a flat window

4.1 Flow Simulation Set-up

The computational setup corresponding to the experiment of Gordeyev et al. [19, 21] is shown schematically in Figure 4.2. The elevation angle of the optical beam is fixed at 120° . The computed flow variables are normalized to characteristic quantities of the free-stream inflow. The reference velocity scale, density and temperature are the free-stream inflow values of a_∞ , ρ_∞ and T_∞ , respectively. The reference length

scale is taken to be the turret radius R . The domain size is $12.3R$, $2.81R$ and $0.8432R$, in the streamwise, vertical and spanwise directions, respectively. The origin of the computational domain is at $2.75R$ upstream the turret axle.

4.1.1 Boundary Conditions

The time-dependent inflow data was generated by a separate wall-modeled LES using the rescale-and-recycle technique described in Section 3.1.1. The inlet boundary-layer thickness δ_{in} is approximately $0.1378R$ and the momentum-thickness Reynolds number is $Re_\theta = 8500$. The mean velocity profile of the collected inflow is depicted in Figure 4.3. The Reynolds number based on the free-stream inflow speed and turret radius is $Re_R = 6.3 \times 10^5$. Free-stream conditions at $M_\infty = 0.5$ are imposed on the top boundary. The top and outlet boundaries are preceded by sponge layers as explained in 3.1.1. The thicknesses of the top and outlet sponge layers are $0.5R$ and $2.0R$, respectively. The bottom boundary is wall modeled with adiabatic boundary conditions. The wall-layer thickness is selected to be close 5% of the inlet boundary-layer thickness and contains 5 mesh points in the wall-normal direction. Periodic boundary condition is applied in the spanwise direction.

4.1.2 Spatial and Temporal Resolutions

The mesh sensitivity of the flow and aero-optical computations using wall-modeled LES was examined by considering three levels of mesh resolution. Unity CFL number was applied for time marching. Table 4.1 lists the mesh sizes, grid spacings at inlet in terms of wall units and time step sizes normalized by outer flow scales. The streamwise mesh spacing at the flat window is $\Delta x = 4.6 \times 10^{-3}R$, $3.6 \times 10^{-3}R$ and $3.0 \times 10^{-3}R$, for meshes A, B and C, respectively. The mesh spacings at the separation point above the sharp edge of the turret is $\Delta x = 3 \times 10^{-3}R$ and $\Delta y_{min} = 1.7 \times 10^{-4}R$, in streamwise and wall-normal directions, respectively. Figure 4.4 displays the mesh

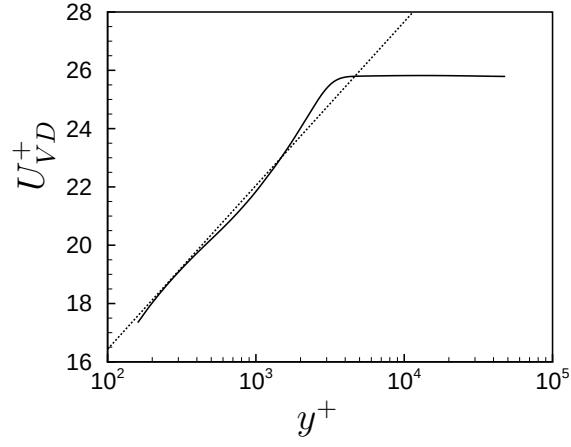


Figure 4.3. Mean streamwise velocity: —, Inflow generated by wall-modeled LES at $M_\infty = 0.5$, $Re_\theta = 8500$; - - - -, logarithmic law $U^+ = (1/0.41) \ln y^+ + 5.2$.

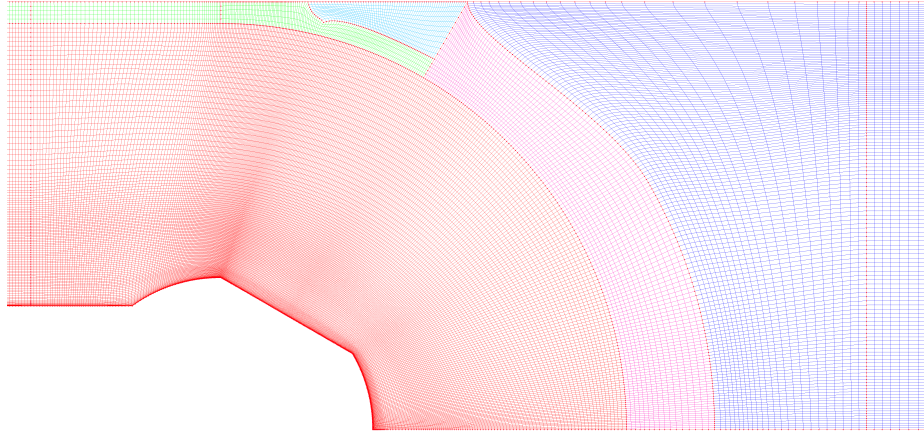


Figure 4.4. A spanwise cut of mesh A surrounding the cylindrical turret showing the mesh topology represented by grid blocks.

TABLE 4.1

PARAMETERS OF THE MESH CONVERGENCE STUDY FOR SUBSONIC
FLOW OVER A CYLINDRICAL TURRET

Mesh	Description	Δx_{in}^+	Δz_{in}^+	$\Delta t \frac{U_\infty}{R}$	Mesh Size	$T_{\text{stats}} \frac{U_\infty}{R}$
A	Coarse Mesh	450	225	7.78×10^{-5}	8×10^6	70
B	Intermediate Mesh	300	150	6.87×10^{-5}	22×10^6	62
C	Fine Mesh	200	100	5.86×10^{-5}	63×10^6	53

NOTE: $\Delta y_{min,inlet}^+ = 30$

Domain Size: $L_x = 12.3R$, $L_y = 2.81R$ and $L_z = 0.8432R$

topology used for all three level of mesh resolution. The mesh cells were designed to be nearly-orthogonal in the regions of separated shear layers and optical beam to ensure that the computation of convective fluxes employs low-dissipative scheme.

In Sections 4.2 and 4.3, the flow and aero-optical statistics were collected on meshes A, B and C for time periods T_{stats} which are also listed in Table 4.1. T_{stats} is sufficiently long to cover at least 4 flow-through-times based on the streamwise domain size and free-stream speed.

4.2 Mesh Convergence Study for Flow Predictions

Four streamwise locations are indicated in blue dashed line at $x/R = 3.00, 3.25, 3.50$ and 3.75 as shown in Figure 4.2 where the flow results are compared to the experimental measurements of Godeyev et al. [21] and previous results from wall-resolved LES (WRLES) [66] conducted at inflow $Re_\theta = 1400$, which equals to 16% of the Reynolds number the inflow of the current simulations.

As shown in Figures 4.5 and 4.6. Reasonable agreement is observed among the current results, the experimental data and previous numerical results. The predicted velocity statistics from wall-modeled LES match the previous results at a reduced Reynolds number [66], which indicates that due to turbulent separation the velocity field in the separated shear layer is insensitive to Reynolds number. There is a small numerical under-prediction in the mean velocity profiles in the upper part of the separated shear layer compared to the experimental results due to the confinement effect of the wind-tunnel walls. The velocity fluctuation levels are over-predicted by 25% compared to experimental measurements especially at streamwise location close to the separation point. The thickness of the shear layer is also over-predicted by the numerical simulations.

Figures 4.7, 4.8 and 4.9 display the predicted rms of the density, pressure and temperature fluctuations, respectively, at the four streamwise locations. The levels of the density fluctuations at the full experimental Reynolds number are lower to the levels of density fluctuations predicted by wall-resolved LES [67] at a reduced Reynolds number. In contrast to turbulent boundary layers, the pressure and density fields have the same levels of fluctuations inside the separated shear layer. The levels of the temperature fluctuations are approximately half of that of density and pressure and get stronger in the downstream direction. This is consistent with the findings of Fitzgerald and Jumper [15] that strong pressure variations inside coherent vortical structures has a dominate effect on the density fluctuations and hence optical distortions, whereas the temperature fluctuations play a less important role than pressure fluctuations.

Despite the clear tendency of mesh convergence of the flow quantities, the rms of density fluctuations in the shear layer are more mesh demanding to converge compared to that of streamwise velocity, especially at streamwise locations far downstream from the separation point at the upper edge of the flat window.

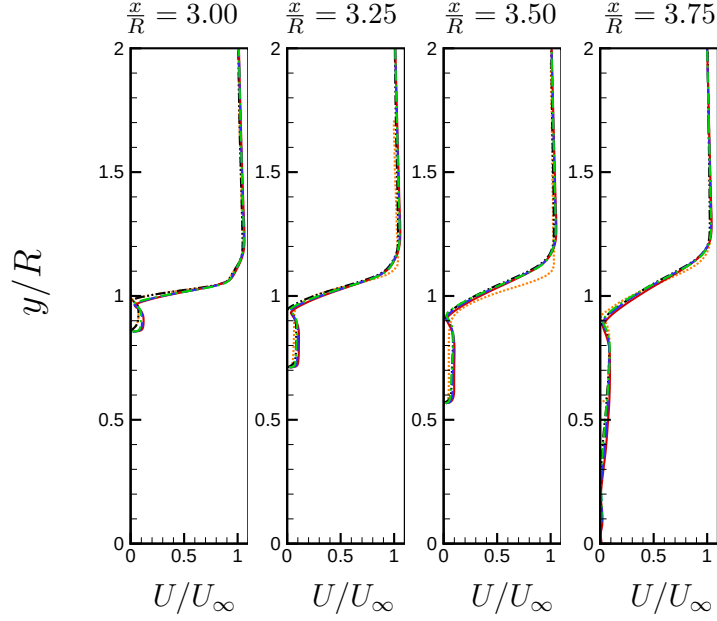


Figure 4.5. Mean velocity profiles above the optical window: wall-modeled LES: —, mesh A; — —, mesh B; — · —, mesh C. — · —, wall-resolved LES at $Re_R = 8.8 \times 10^4$ [67]. - - - -, experiment of Gordeyev et al. [21].

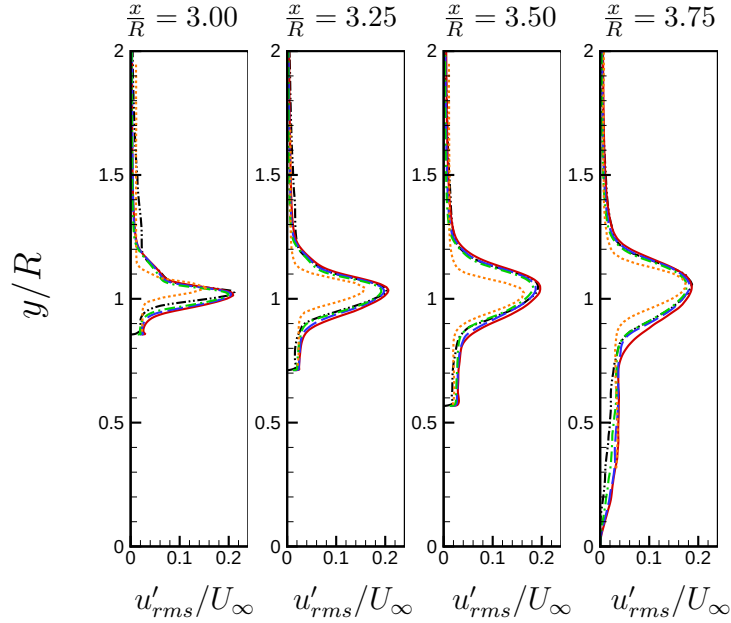


Figure 4.6. Root-mean-square of velocity fluctuations above the optical window: wall-modeled LES: —, mesh A; — —, mesh B; — · —, mesh C. — · —, wall-resolved LES at $Re_R = 8.8 \times 10^4$ [67]. - - - -, experiment of Gordeyev et al. [21].

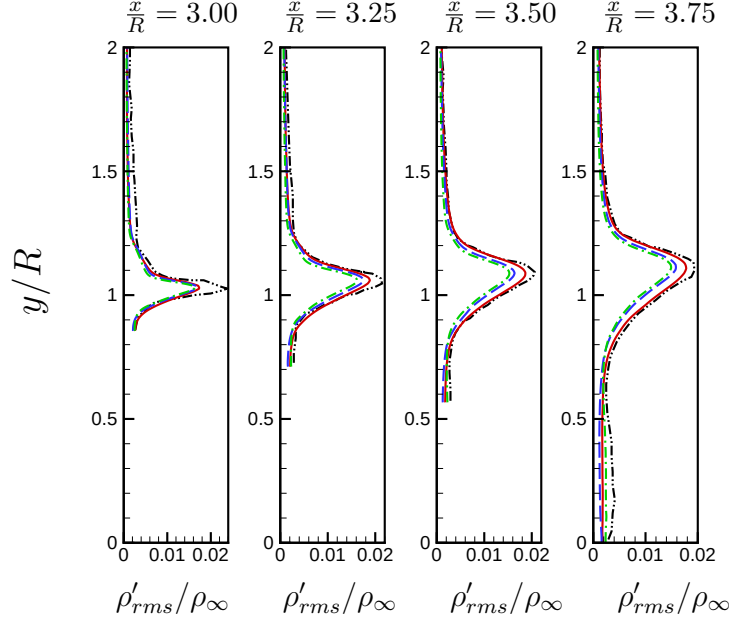


Figure 4.7. Root-mean-square of density fluctuations above the optical window:
 wall-modeled LES: —, mesh A; — — —, mesh B; — · — · —, mesh C. — · · — · —, wall-resolved LES at $Re_R = 8.8 \times 10^4$ [67].

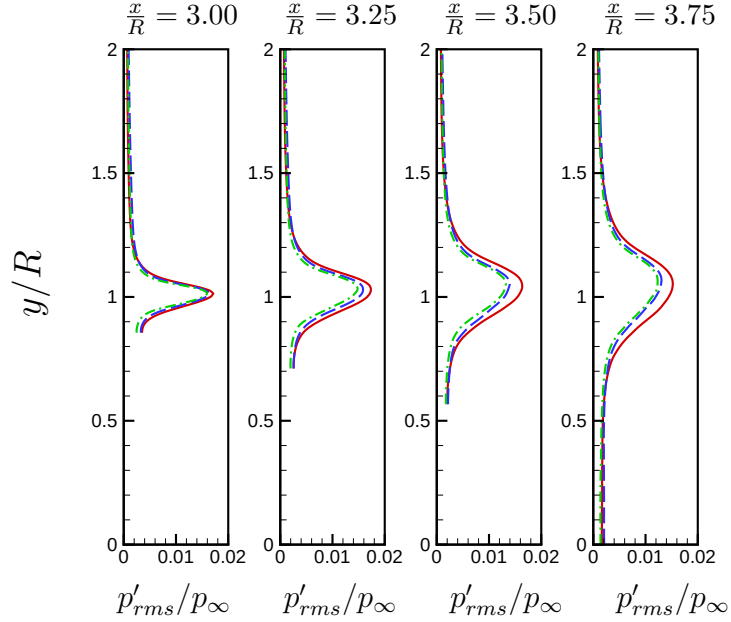


Figure 4.8. Root-mean-square of pressure fluctuations above the optical window:
 —, mesh A; — — —, mesh B; — · — · —, mesh C.

4.3 Mesh Convergence Study for Aero-Optical Prediction

As shown in Figure 4.2, the instantaneous density field was collected every 16 time steps on the beam grid, whose local coordinates is x' , y' and z' , in the local stream-wise, beam-propagation and spanwise directions, respectively. The origin of the local axes is located at the aperture center at $(x_c, y_c, z_c) = (3.18R, 0.75R, 0.4216R)$. The resolution of the beam grid is close to that of the LES grid at each level of mesh refine. The grid covers the whole span, and its height is $2R$. The aero-optical calculations are performed with an aperture of size $A_{x'} \times A_{z'} = R \times 0.8432R$. Figure 4.10 displays an instantaneous piston-tip-tilt-removed wavefront on the fine mesh (C). It illustrates the effect of coherent large vortical structures on the wavefront distortions.

In Table 4.2, optical distortions of a reduced aperture size $0.75R \times 0.8432R$ is considered for consistent comparison to the experimental measurements [19] of aperture size $0.75R \times 0.875R$ using 2-D Shack-Hartmann wavefront sensor and wall-resolved LES [66] (case D) at a reduced Reynolds number $Re_R = 8.8 \times 10^4$ and aperture size of $0.75R \times 0.78R$. There is a tendency of mesh convergence of the magnitude of optical distortions: the differences between meshes A and B and between B and C are 19% and 10%, respectively. The predicted normalized $\overline{\text{OPD}}_{rms}$ of mesh C is 25% less than the experimental results using 2-D wavefront sensors. The value of the normalized $\overline{\text{OPD}}_{rms}$ decreases with mesh refinement and has more negative difference compared to the experimental measurements using 2-D wavefront sensors. This trend agrees with the predicted optical distortions by Mathews et al. [44] for a high-Reynolds-number flow over a hemisphere-over-cylinder turret at different levels of mesh resolution using the same flow solver. Additional work is needed to address the issue of discrepancy in $\overline{\text{OPD}}_{rms}$ between 2-D wavefront measurements and numerical predictions using relatively well-resolved mesh.

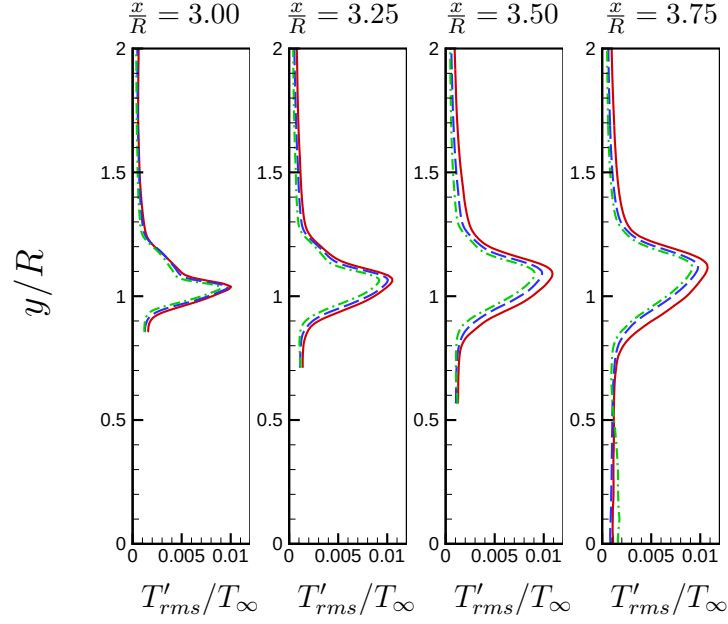


Figure 4.9. Root-mean-square of temperature fluctuations above the optical window: —, mesh A; - - -, mesh B; - · -, mesh C.

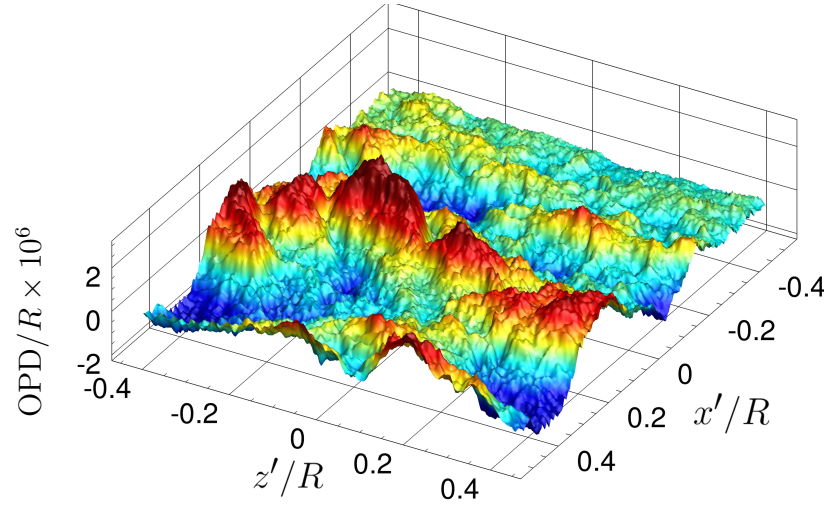


Figure 4.10. Instantaneous piston-tip-tilt-removed wavefront on the fine mesh (C) for aperture size $A_{x'} \times A_{z'} = R \times 0.84R$.

TABLE 4.2

MAGNITUDE OF OPTICAL DISTORTIONS FOR SUBSONIC FLOW OVER A
CYLINDRICAL TURRET

Description	Re_R	$A_{x'} \times A_{z'}$	$\frac{\overline{\text{OPD}}_{rms}}{(\rho_\infty/\rho_{SL})DM^2}$
Wall-modeled (A)	6.5×10^5	$0.75R \times 0.84R$	1.49×10^{-6}
Wall-modeled (B)	6.5×10^5	$0.75R \times 0.84R$	1.25×10^{-6}
Wall-modeled (C)	6.5×10^5	$0.75R \times 0.84R$	1.12×10^{-6}
Wall-resolved LES [66]	8.8×10^4	$0.75R \times 0.78R$	1.48×10^{-6}
Exp. measurements [19, 21]	6.5×10^5	$0.75R \times 0.88R$	1.50×10^{-6}

The two-point correlation of wavefront distortions as defined by Eq.(3.2), is calculated for the three meshes. The iso-contours of $\mathcal{R}_{\text{OPD}}$ are depicted in Figures 4.11(a), (b) and (c) for meshes A, B and C, respectively. Also Figures 4.12(a) and (b) present the correlations of OPD at the aperture center in the streamwise and spanwise directions, respectively. The correlation lengths of OPD in the streamwise and spanwise directions from the wall-modeled LES are larger than those from wall-resolved LES at a reduced Reynolds number [67]. As more small-scale structures are well-resolved by mesh refinement, the structures of OPD becomes less correlated in streamwise and spanwise directions.

The density flow structures, which have a direct connection with the optical wavefront structures, can be illustrated by two-point correlation of density fluctuations as defined by Eq.(3.6). Figures 4.13(a), (b) and (c) display the iso-contours of $\mathcal{R}_{\rho\rho}$ at 3 points close to the center-line of the shear layer at three different streamwise locations: $(x/R, y/R) = (3.04, 1.0), (3.33, 1.0)$ and $(3.62, 1.0)$, respectively, for the three level of mesh resolutions A, B and C. It is shown that the correlation length in

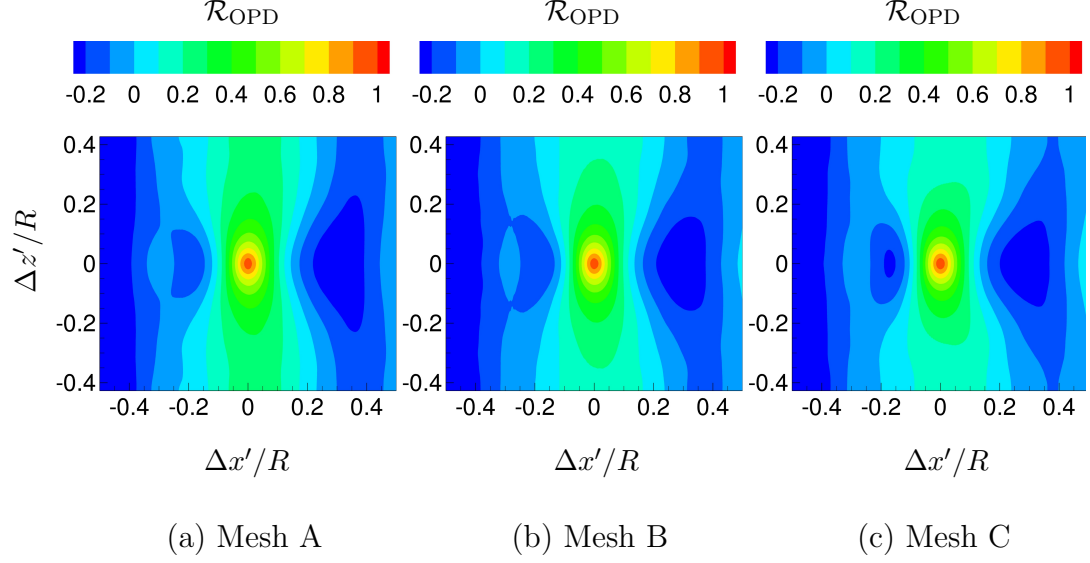


Figure 4.11. Two-point correlation coefficient of OPD at the aperture center.

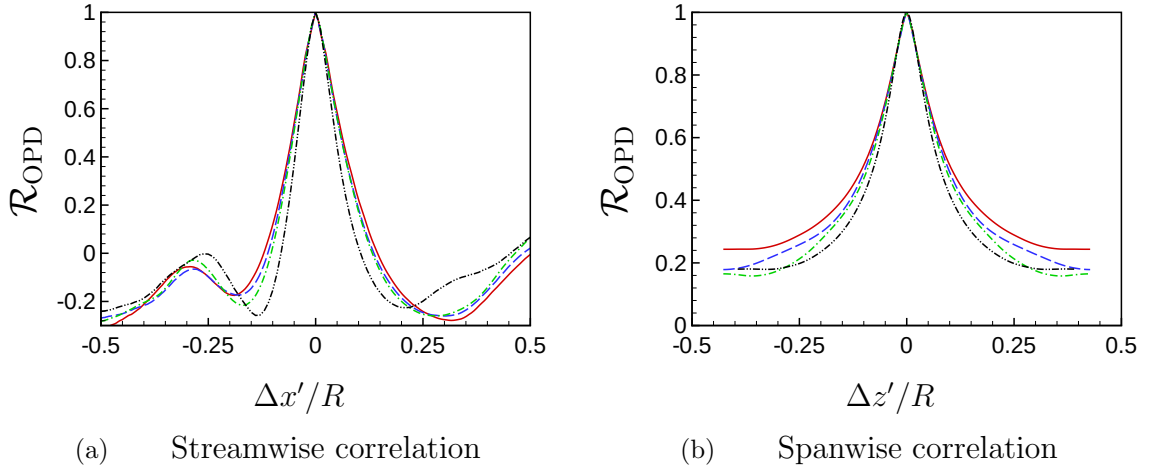


Figure 4.12. Correlation coefficient of OPD at the aperture center. Wall-modeled LES: —, mesh A; - - -, mesh B; - · - · -, mesh C. · · · · -, wall-resolved LES at $Re_R = 8.8 \times 10^4$ [67]

both streamwise and cross-stream direction increase due to the growth of separated shear layer. The strong negative correlation on both upstream and downstream sides indicates the dominance of large-scale coherent vortical structures in the shear layer. The mesh refinement leads to the shrinkage of these regions because of resolving more small-scale structures.

Figures 4.14(a), (b) and (c) demonstrate the spatial correlations of density fluc-

tuations in the beam-propagation direction (i.e. y' -direction) at the aforementioned points close the center-line of the separated shear layer. They show that the correlation lengths of density structures are almost grid-insensitive in the beam-propagation direction. Based on Sutton's linking equation 3.7, this indicates that grid sensitivity of predicted optical distortions, in terms of $\overline{\text{OPD}}_{rms}$, is mostly affected by grid sensitivity of density fluctuations as shown in Figure 4.7.

Figure 4.15 depicts the space-time correlation of OPD at the aperture center, as defined in Eq.(3.3.2), for the case of fine mesh (C). It is noted that the space-

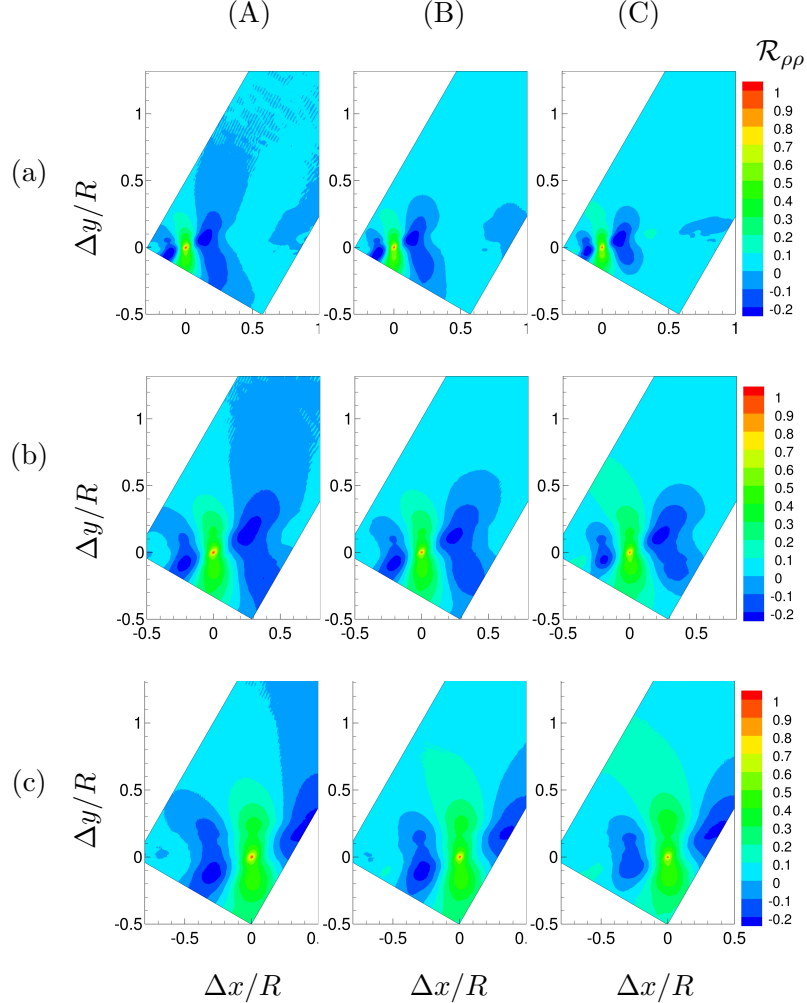


Figure 4.13. Two-point correlation coefficient of density fluctuations at locations: (a) $y/R = 1.0, x/R = 3.04$; (a) $y/R = 1.0, x/R = 3.33$; (c) $y/R = 1.0, x/R = 3.62$; (A) coarse mesh; (B) intermediate mesh; (C) fine mesh.

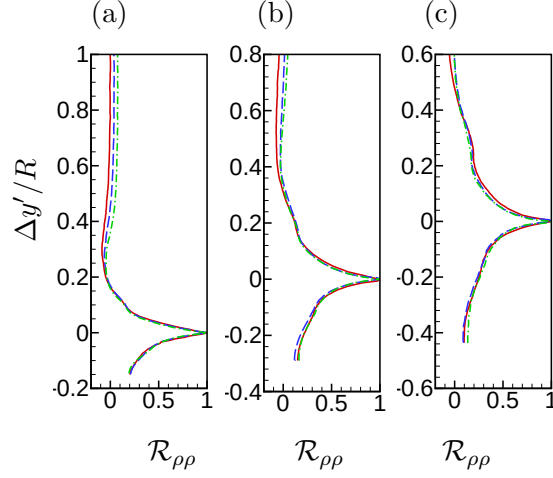


Figure 4.14. Wall-normal space correlation coefficient of density fluctuations at different locations: (a) $y/R = 1.0, x/R = 3.04$; (b) $y/R = 1.0, x/R = 3.33$; (c) $y/R = 1.0, x/R = 3.62$. —, mesh A; - - -, mesh B; - · -, mesh C.

time correlation is not symmetric with respect to the origin. The correlation in the positive space and time separations are larger than that in the negative space and time separations, as a result of the shear layer development.

Figure 4.16 displays the frequency power spectral density of OPD at the aperture center for the three meshes, and a convergence trend is observed by mesh refinement.

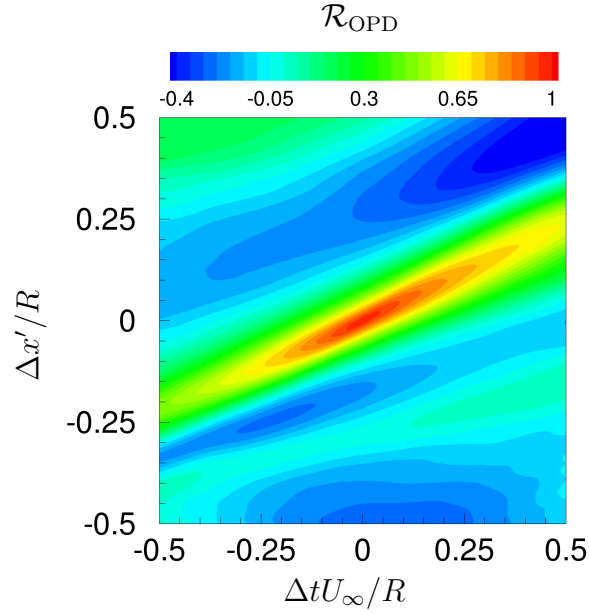


Figure 4.15. Space-time correlation of OPD at the aperture center on mesh C.

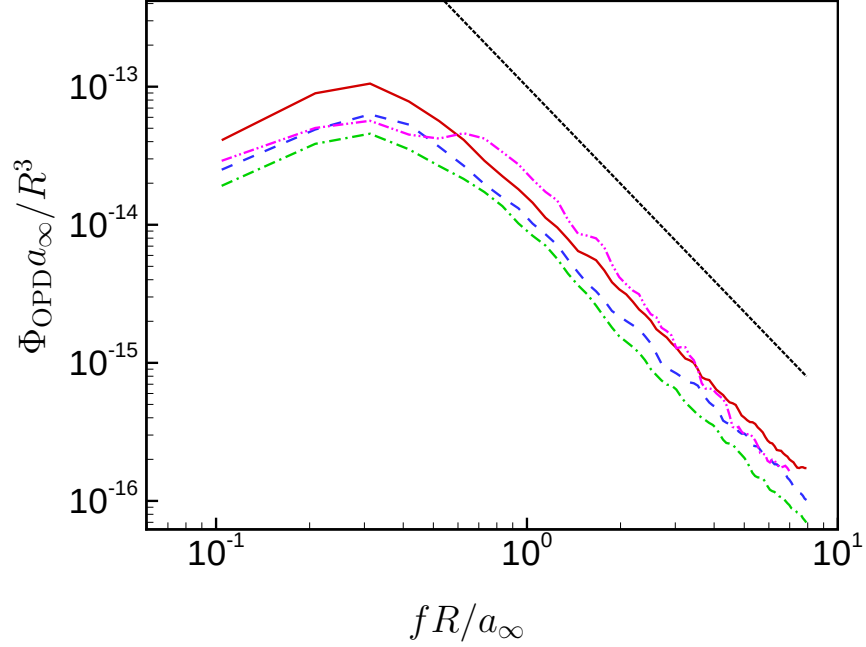


Figure 4.16. Frequency spectra of OPD at aperture center. Wall-modeled LES: —, mesh A; - - -, mesh B; - · - · -, mesh C. · · · · -, wall-resolved LES at $Re_R = 8.8 \times 10^4$ [66]; - - - -, line with $-7/3$ slope.

the spectra exhibits a prominent inertial subrange, which is consistent with the previous observation in the wall-resolved LES at a reduced Reynolds number [66]. The spectra roll off at a rate of $-7/3$, which is slightly slower than that in the wall-resolved LES.

4.4 Conclusions

Wall-modeled LES is employed to compute subsonic flow over a cylindrical turret in the same flow configuration as the experiment conducted by Gordeyev et al. [19, 21] at Mach number $M_\infty = 0.5$ and turret-radius Reynolds number $Re_R = 6.5 \times 10^5$. The elevation angle of the optical beam is fixed at $\theta = 120^\circ$ with respect to the upstream. Simulations are conducted at three levels of mesh resolution to examine the grid sensitivity of flow and aero-optical predictions. Although there is a clear tendency towards mesh-convergence for both flow and optical results, the levels

and structures of density fluctuations, and hence optical distortions are more mesh-demanding compared to those of velocity fluctuations, as in the case of turbulent boundary layers. Pressure and density fluctuations have similar magnitudes, and are stronger than the temperature fluctuations, in contrast to those in turbulent boundary layers. This is consistent with the conclusion of Fitzgerald and Jumper [15] that pressure fluctuations play a dominate role in inducing the density fluctuations and optical distortions. The wall-modeled LES underpredicts the magnitude of the wavefront distortions, but captures their spatio-temporal characteristic reasonably well.

CHAPTER 5

TRANSONIC FLOWS OVER A CYLINDRICAL TURRET

Numerical predictions of the aero-optical effects of they cylindrical turret flow were extended the transonic flow regime. The flow configuration in the experiment conducted by Vorobiev at al. [63] was considered. Wall-modeled LES is validated based on the steady and unsteady pressure measurements on the turret surface and the wind-tunnel wall, and the aero-optical predictions were compared to the two-dimensional Shack-Hartmann wavefront data. The LES captures the low-frequency oscillations of the shockwave as it interacts with the separated shear layer and wake, and provides a better understanding of the evolution of the periodic shock-wake interaction. Phase averaging based on the fundamental frequency of the shock wandering was employed to study the flow field and optical distortions and the underlying physics.

5.1 Flow Configuration

The experiments of Vorobiev et al. [63] were conducted at the transonic in-draft wind-tunnel facility at the Hessert Laboratory for Aerospace Research at University of Notre Dame. A partially-protruding cylinder with a conformal window was mounted in a wind-tunnel test section with a height jump at the location of the cylinder axle. The inlet cross-section is $100 \times 100 \text{ mm}^2$ and the turret spans the whole width. In the experimental setup, the test section downstream the cylindrical turret is followed by a 3-D dimensional diffuser. The expansion angles of the diffuser are 5.1° and 6.1° in the vertical and spanwise directions, respectively. Two vacuum pumps were used

with different valve settings to control the outlet back pressure, and consequently the incoming flow speed. The turret has a square aperture $30 \times 30 \text{ mm}^2$ with its outer cylindrical lens having the same curvature as that of the turret surface. As shown in Figure 5.1(a), nine static pressure ports, marked by P1 to P9, were placed on the side wall of the test section, to measure the pressure distribution along the test section along with two Kulite unsteady pressure ports K10 and K11. The elevation angle (θ) of the beam is measured from the upstream.

As displayed in Figure 5.1(b), nine Kulite unsteady pressure sensors, notated by K1 to K9, were distributed on the turret surface in a staggered pattern with angular step of 3.67 degrees in a range from -14.67 to 14.67 degrees from the middle of the aperture. The elevation angle at the aperture center is $\theta = 105^\circ$.

The flow field surrounding the turret surface was recored by high-speed shadow-graph system, which is synchronized with the unsteady pressure measurements and wavefront measurements using a high-speed 2-D Shack-Hartmann sensor.

By reducing the outlet back pressure, the inflow Mach number increases. Six different test cases were considered and numbered from 1 to 6 in the order of decreasing back pressures. Based on the pressure measurements at probes P1 to P9, the local free-stream Mach number (M_∞) was calculated via the isentropic relation by assuming a constant total pressure along the test section. Figure 5.2 depicts the Mach number distributions for different flow regimes along the wind-tunnel section as a function of the streamwise location relative to the turret center, $(x - x_t)/D$, where x_t and D are the turret center streamwise location and turret diameter, respectively. The flow is locally supersonic above the turret top. The test cases numbered from 1 to 6 correspond to inlet Mach numbers from 0.65 to 0.68. In cases 4, 5 and 6, the flow was choked at inlet Mach number $M_{\infty,in} = 0.68$. Maximum levels of turret-surface pressure fluctuations were achieved in cases 3 and 4 indicating the strongest shock oscillations.

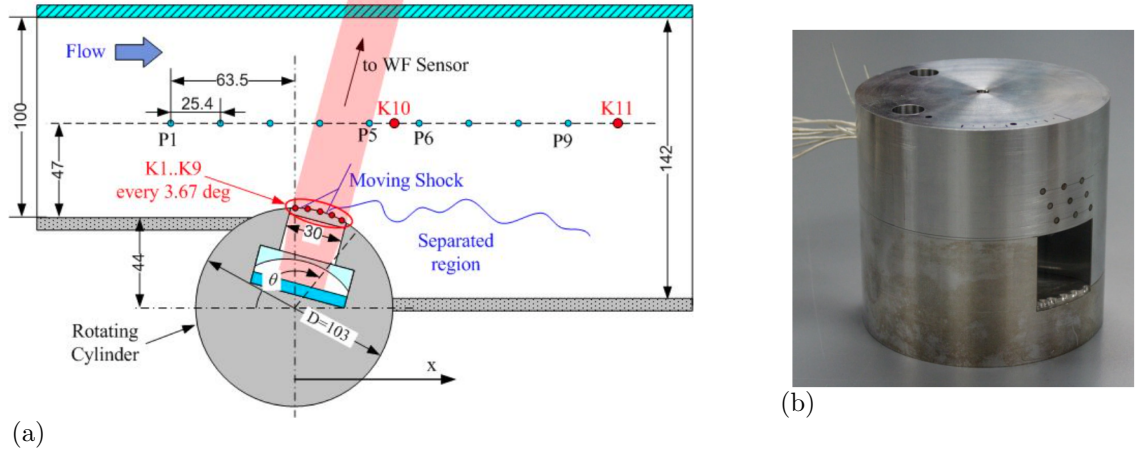


Figure 5.1. (a) Schematic of the experimental setup of Vorobiev et al. [63]. Dimensions are in millimeters. (b) Photo of the turret with unsteady pressure sensors [63].

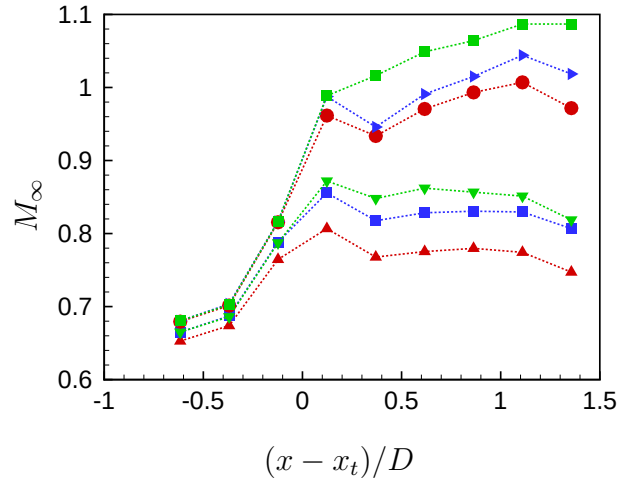


Figure 5.2. Mach number along the test section for different flow cases from the experiment of Vorobiev et al. [63]: --▲--, case 1; --■--, case 2; --▼--, case 3; --●--, case 4; --▶--, case 5; --■--, case 6.

5.2 Flow Simulation Set-up

The computational setup of the flow configuration is shown schematically in Figure 5.3. The experimental test case 4 was considered in the numerical simulations because it has well-defined inlet boundary conditions represented by choked inflow at $M_{\infty, in} = 0.68$ and $Re_{\theta, in} = 10^4$ and is considered one of the cases characterized by strong shock oscillations. However, The outlet boundary condition of all test cases is not well-defined due to the lack of experimental measurements of pressure field far downstream the turret. This will be discussed in more details in 5.2.3. Time histories of the flow variables were collected at the P and K probes locations as in the experimental setup.

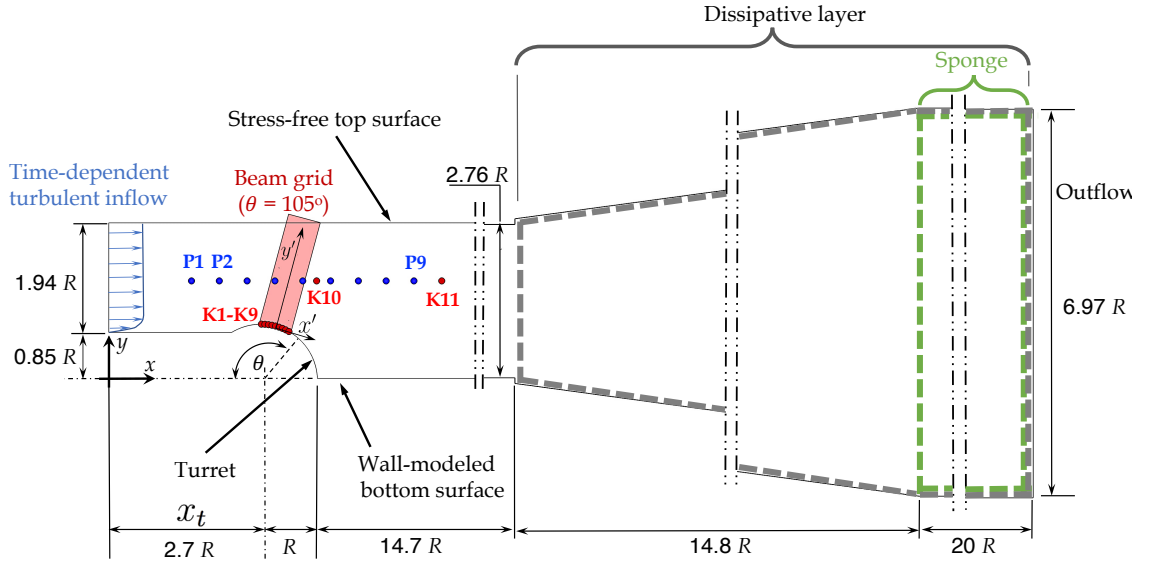


Figure 5.3. Computational setup of the numerical simulation of the transonic flow over a cylindrical turret.

The origin of the coordinate system is located $2.7R$ upstream the turret axle. The

total size of the computational domain is $53.2R$, $6.97R$ and $0.2R$, in the streamwise, vertical and spanwise directions, respectively. The elevation angle of the optical beam is fixed at 105° .

Since the spanwise domain size is constant in the LES, whereas in the experiment the diffuser is three-dimensional, the slope of the diffuser walls in the simulation is increased such that the expansion rate of the cross sectional area is the same as in the 3-D diffuser.

5.2.1 Spatial and Temporal Resolutions

The mesh size is 34 million grid points. There are 64 equally-spaced cells along the span with $\Delta z = 3.125 \times 10^{-3}R$. At the inlet, the mesh spacings are $\Delta x = 0.1\delta_{in}$, $\Delta y_{min} = 0.75 \times 10^{-2}\delta_{in}$ in streamwise and wall-normal directions, respectively, where $\delta_{in} = 0.065R$ is the thickness of incoming turbulent boundary layer. The mesh resolution over the optical aperture is $\Delta x = 2.5 \times 10^{-3}R$ in the streamwise direction and $\Delta y_{min} = 5 \times 10^{-4}R$ in the wall-normal direction for $\theta = 90^\circ$ to 120° . The mesh topology is divided into a number of grid blocks, most of which are structured and consist only of hexahedra. Only two blocks located downstream and above the turret mesh block consist of a blend of hexahedra and prisms to ensure good mesh quality in the regions of interest.

5.2.2 Shock Capturing Method

CharLES uses a hybrid Central-ENO scheme to simulate flows involving shocks. The solver computes the convective fluxes based on three main parts: a second-order central scheme with upwinding factor for less-than-perfect mesh quality, a second-order ENO to perform left- and right-biased flux reconstructions and a detector for the shock location to switch from central scheme to ENO scheme for the convective fluxes reconstruction across the shock. The shock detector proposed by Bhagatwala

and Lele [5] for a modified form of Ducros shock sensor [13] is used in the present study. It can be written as:

$$C_{sc} = \frac{1}{2} [1 - \tanh(2.5 + 5 \tau_c \nabla \cdot \mathbf{u})] \frac{\|\nabla \cdot \mathbf{u}\|^2}{(\|\nabla \cdot \mathbf{u}\|^2 + \|\nabla \times \mathbf{u}\|^2 + \epsilon)}, \quad (5.1)$$

where C_{sc} is the shock-capturing coefficient. Shock-capturing using ENO scheme is activated if $C_{sc} > 0.9$. This switch employs two sensors: the first sensor, represented by the term in the square brackets, acts as a switch that turns off for positive dilatation, whereas it rises smoothly as the negative dilatation increases. Therefore this switch turns on only when there is significant compression in the flow. The second sensor, represented by the fraction, is based on relative magnitudes of dilatation and vorticity as introduced by Ducros et al [13]. τ_c is a characteristic time scale for the normalization of dilatation and it is set to be $\tau_c = 5R/a_{\infty, in}$. ϵ is a positive real number chosen to prevent singularity in regions where both dilatation and vorticity magnitudes are zero, and is set to be 10^{-7} .

5.2.3 Boundary Conditions

Periodic boundary condition was applied in the spanwise direction. Time-dependent boundary-layer inflow, at $M_{\infty, in} = 0.68$ and $Re_{\theta, in} = 1.0 \times 10^4$, is generated by an independent simulation using the rescale-and-recycle technique as described in Section 3.1.1. The mean velocity profile of the inflow is depicted in Figure 5.4.

The bottom wall and turret surface are wall-modeled with adiabatic boundary conditions. The wall-layer thickness at the turret surface is $2.5 \times 10^{-3}R$ and contains 5 mesh points in the wall-normal direction. The top boundary is assigned to be impermeable and stress-free in order to minimize the mesh resolution requirement in the top region, which is of less interest compared to the regions near the bottom wall and turret surface. The flow is computed using fully-upwind dissipative scheme

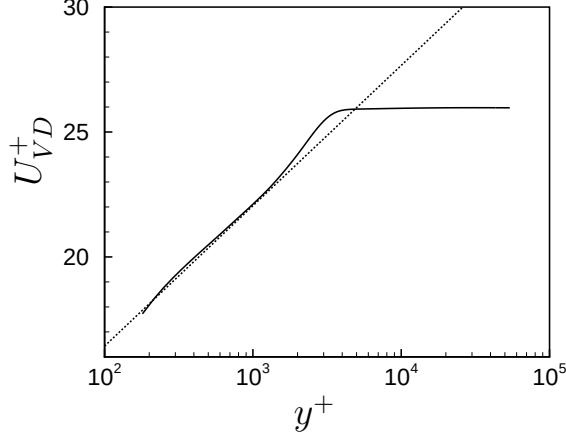


Figure 5.4. Mean streamwise velocity: —, inflow generated by wall-modeled LES at $M_\infty = 0.68$, $Re_\theta = 1.0 \times 10^4$; - - - -, logarithmic law $U^+ = (1/0.41) \ln y^+ + 5.2$.

only in the range from $x = 18.4R$ downstream the turret axle to the outlet boundary to damp out the vortical structures and acoustic waves before reaching the outlet boundary. In addition to this dissipative layer, the outlet boundary is preceded by a sponge layer, as explained in Section 3.1.1, with a thickness of $20R$.

Due to the lack of experimental measurements far downstream the turret, the back pressure assigned to the outlet was determined by an iterative procedure to match the mean pressure at the K-probes on the turret surface in the experimental case 4.

If $P_{\text{back},1}$ and $P_{\text{back},2}$ are the outlet pressures assigned for two different realizations 1 and 2, respectively, the back pressure that will be assigned in the third iteration ($P_{\text{back},3}$) is determined by:

$$P_{\text{back},3} = P_{\text{back},1} + f(P_{\text{back},2} - P_{\text{back},1}) \quad (5.2)$$

The interpolation factor (f) was determined by the least-squares of the following

set of equations written in index notation:

$$\begin{aligned} [P_2 - P_1]_K f &= [P_{exp,4} - P_1]_K, \\ a_K f &= b_K, \end{aligned} \quad (5.3)$$

where K notates for the K -probe index ranging from $K = 1$ to $K = 9$. P_1 and P_2 are the mean pressure at a probe K at the realizations 1 and 2, respectively. $P_{exp,4}$ is the mean pressure measured at a probe K in the test case 4. Equation(5.4) can be rewritten in the vector form as:

$$\mathbf{a}f = \mathbf{b}. \quad (5.4)$$

Thus the factor f is determined by a least-square method to match the experimentally measured mean pressure at most of the turret surface probes:

$$f = \frac{\mathbf{a}^\top \mathbf{b}}{\mathbf{a}^\top \mathbf{a}}. \quad (5.5)$$

The flow fields caused by back pressures $0.84P_{in}$, $0.94P_{in}$ and $1.078P_{in}$ are shown in Figures 5.5(a),(b) and (c), respectively, using the numerical schlieren (NS), which is defined as:

$$NS = -10 \exp \left(\frac{\|\nabla \rho\| - \|\nabla \rho\|_{min}}{\|\nabla \rho\|_{max} - \|\nabla \rho\|_{min}} \right). \quad (5.6)$$

Strong shock oscillations are observed as the back pressure increases, which is consistent with experimental shadowgraphs displayed in Figures 5.5(d),(e) and (f). After iterations, the back pressure for the outlet boundary conditions is set to be $P_{back} = 1.078P_{in}$, which gives the best possible match of the mean pressure at the turret surface and wind-tunnel wall as shown in Figure 5.6. Due to the positive mass flow rate imposed at the inlet, there will be no global flow reversal even if the outlet back pressure is slightly higher than the inlet pressure.

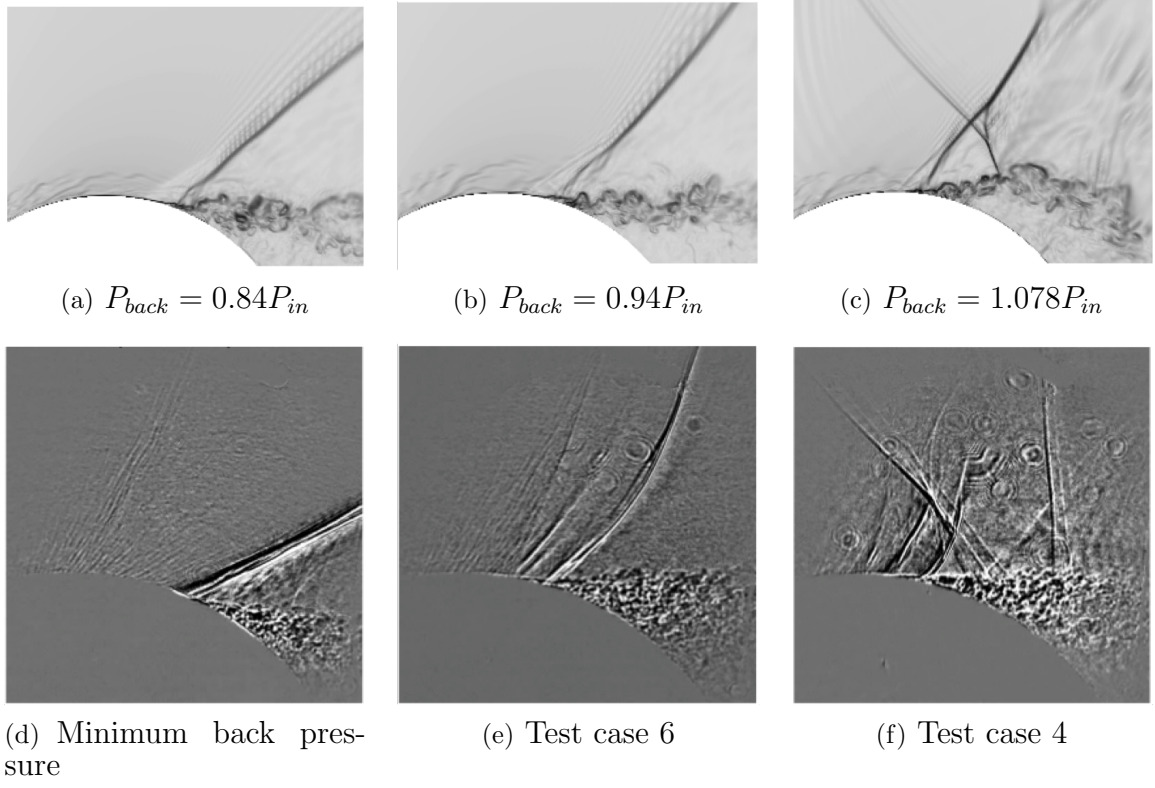


Figure 5.5. A qualitative comparison between numerical schlieren (a,b and c) and experimental shadowgraphs (d,e and f) of Vorobiev et al. [63].

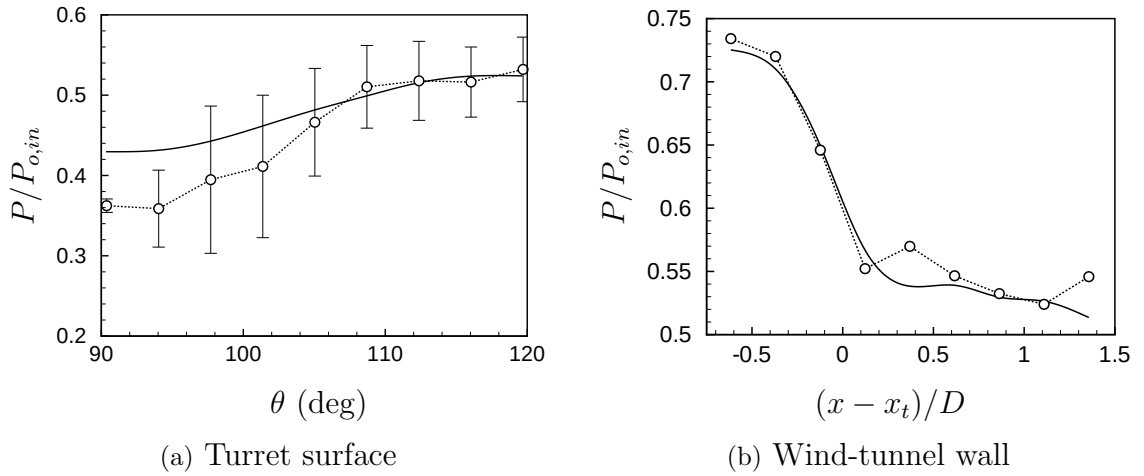


Figure 5.6. Mean pressure distribution: —, wall-modeled LES predictions; - - o - -, experimental measurements of Vorobiev et al. [63] for case 4.

5.3 Flow Field Results

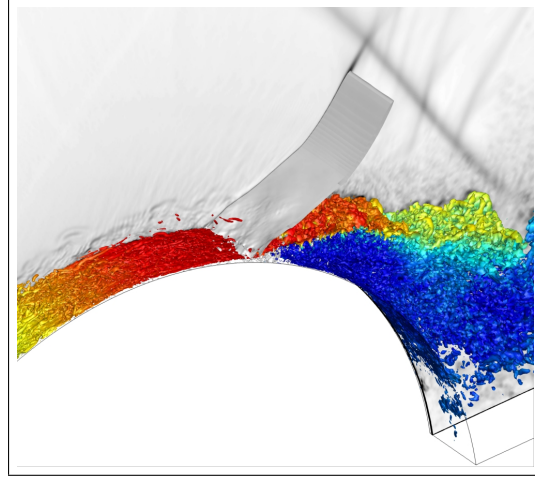
The most prominent characteristic of the flow is the strong cyclic motion of shock-wave with a time period of $t_{cycle} \approx 2.3D/U_{\infty,in}$. Figure 5.7 displays the instantaneous flowfield at different time instants during one cycle. The shock location is identified by the iso-surface of a strong negative dilatation of value $\nabla \cdot \mathbf{u} = -90a_{\infty,in}/R$. The vortical structures are represented by iso-surfaces of λ_2 criterion colored by stream-wise velocity contours. The numerical schlieren, as defined by Eq.(5.6), is displayed in a spanwise-cut plane in the back.

A strong shock moving upstream induces flow separation immediately after the shock. During this upstroke, the separation region enlarges while the shock becomes weaker (Figure 5.7(a–e)). During the downstroke, the flow separation reduces and the shock strengthens when moving downstream. The shock motion and intensity have asymmetric pattern during its upstroke and downstroke.

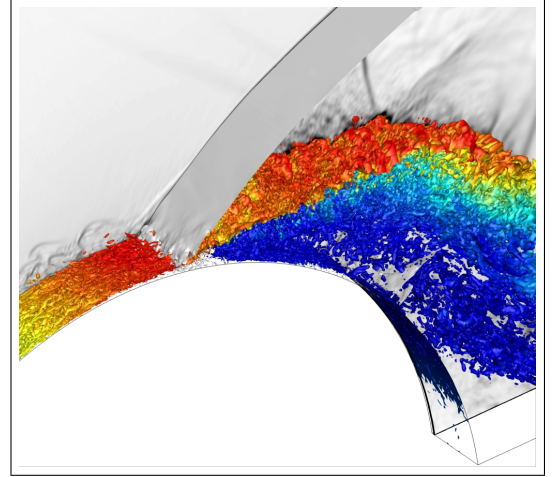
It is illustrated that the key features of shock and flow dynamics are highly coherent in the spanwise direction, which renders spanwise averaging a valid approach for data reduction to study the flow field surrounding the turret, and compare the numerical predictions to experimental measurements.

Figure 5.8(a) depicts the spanwise-averaged instantaneous pressure on the turret surface at a range of elevation angle θ from 90° to 120° covering the optical aperture. It is compared to the experimental measurements of unsteady pressure at the Kulite probes from K-1 to K-9 as shown in Figure 5.8(b). The pressure in both figures is normalized by the inlet total pressure $P_{o,in}$. The predicted unsteady pressure exhibits a large-scale motion with a low frequency of approximately 1 kHz, due to the shock wandering, which is consistent with the experimental observations.

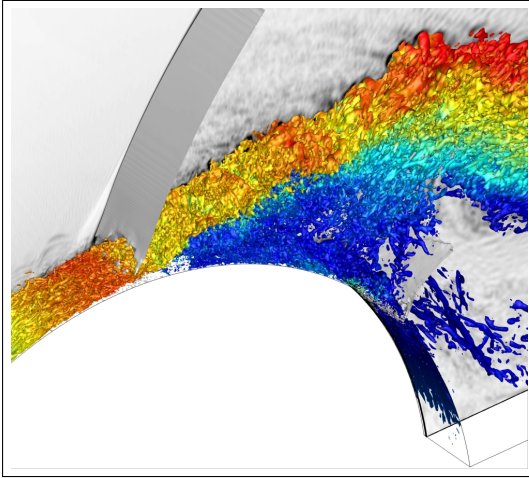
Figure 5.9 illustrates the instantaneous pressure fluctuations, $p' = p - P$. Both sets of numerical and experimental results are depicted on the optical aperture at three elevation angles $\theta = 97.6^\circ, 105^\circ$ and 112.3° . It is shown that they have similar



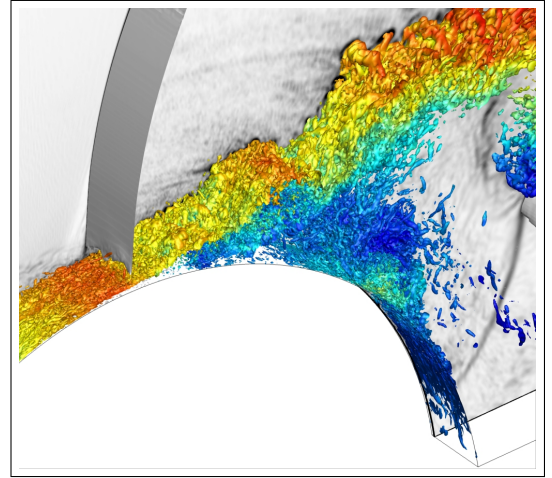
(a) $t = 0$



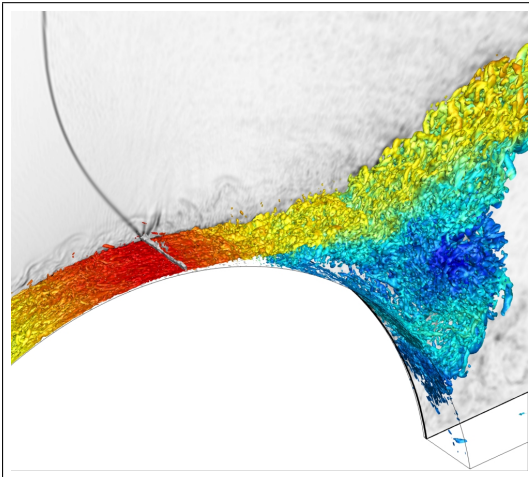
(b) $t = 0.43D/U_{\infty,in}$



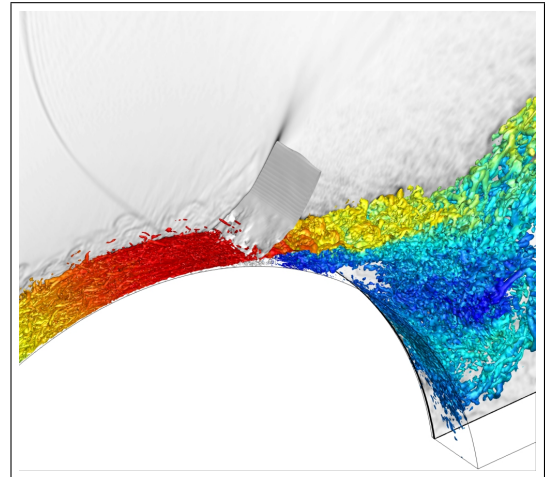
(c) $t = 0.86D/U_{\infty,in}$



(d) $t = 1.29D/U_{\infty,in}$



(e) $t = 1.89D/U_{\infty,in}$



(f) $t = 2.15D/U_{\infty,in}$

Figure 5.7. Instantaneous transonic flow over a cylindrical turret: iso-surfaces of λ_2 criterion colored by streamwise velocity. The grey iso-surface of negative dilatation indicates the shock locus. Contours of numerical schlieren are shown in a spanwise-cut plane in the back.

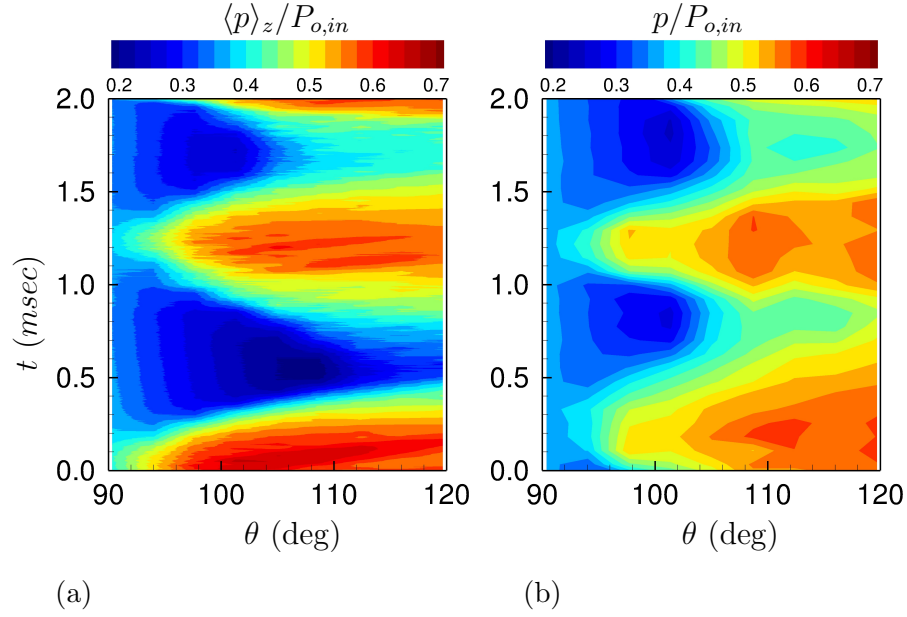


Figure 5.8. Instantaneous pressure over the turret surface: (a) spanwise-averaged pressure from numerical predictions; (b) experimental measurements of case 4 at Kulite probes from K-1 to K-9 (Vorobiev et al. [63]).

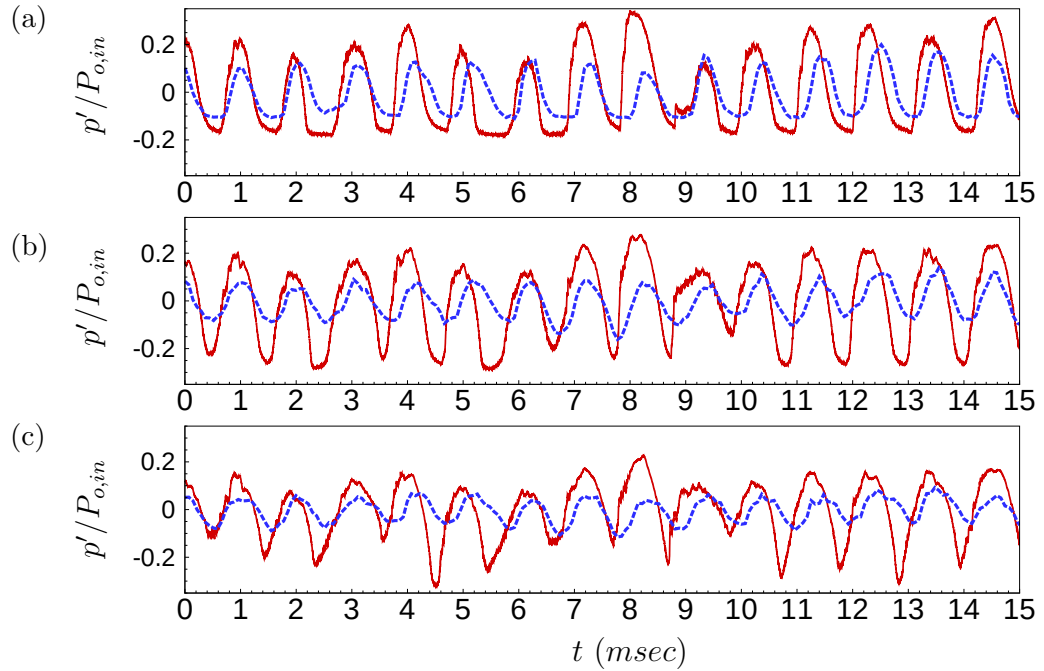


Figure 5.9. Pressure fluctuations at different probes on the turret surface: (a) K-3 probe ($\theta = 97.7^\circ$); (b) K-5 probe ($\theta = 105^\circ$); (c) K-7 probe ($\theta = 112.3^\circ$): —, numerical prediction; — — — —, experimental measurements of case 4 [63].

low frequency content due to the shock oscillations. However, the levels of numerically predicted pressure fluctuations are higher than those of the experimental measurements.

The flow statistics are calculated over a time period $T_{\text{stats}} = 43D/U_{\infty, \text{in}}$. The flow variables are collected at the probe locations shown in Figure 5.3 every 10 time steps, corresponding to a sampling frequency of 2.7 MHz, which is much higher than the sampling frequency of the Kulite measurements (54 kHz) in the experiment of Vorobiev et al.[63].

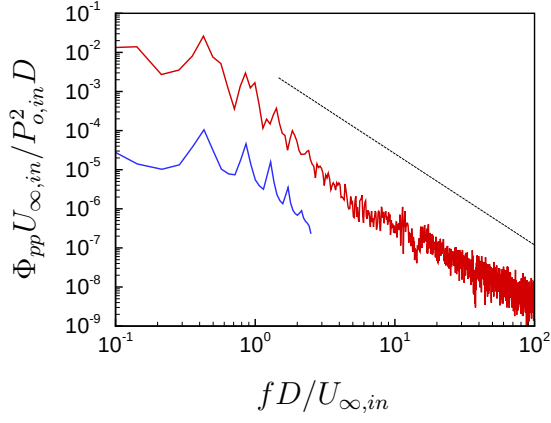
The power spectral density of the pressure fluctuations is compared to the experimental measurements at the Kulite probes. Figures 5.10(a–f) show this comparison at some locations of the Kulite probes over the turret surface and wind-tunnel section. Both the numerical and experimental spectra show a major peak at Strouhal number $St_D = fD/U_{\infty, \text{in}} = 0.426$ and minor peaks at its harmonics indicating a strong periodicity in the flow.

The sampling frequency of the Kulite probes is not sufficiently high to capture the flow scale in the high-frequency end. The roll-off of the numerically-predicted pressure spectra at high frequencies agrees well with the power law of $f^{-7/3}$, especially on the turret surface. However, it decays faster at higher frequencies at K-10 probe where there are no energetic small flow scales, because they are located far away from the separated shear layer.

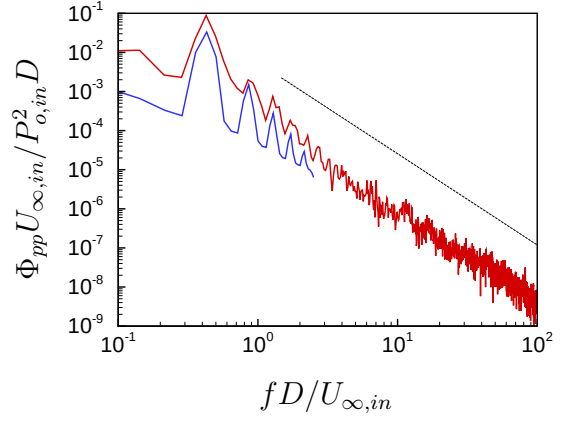
It is noted that the wall-modeled LES greatly over-predicts the magnitude of pressure fluctuations, which may indicate that a finer mesh is required in the near-wall region.

5.4 Aero-Optical Prediction

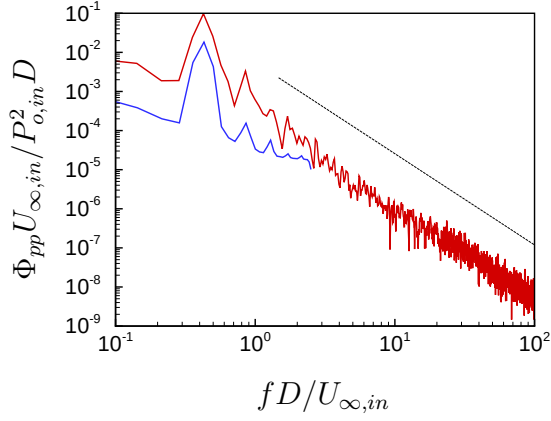
The instantaneous density was interpolated onto the beam grid depicted in Figure 5.3, with dimensions $L_{x'} \times L_{y'} \times L_{z'} = 0.5825R \times 2.02R \times 0.2R$. The grid resolution



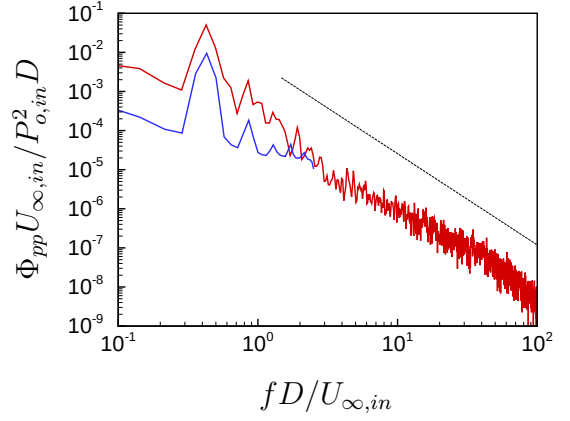
(a) K-1 probe ($\theta = 90.33^\circ$)



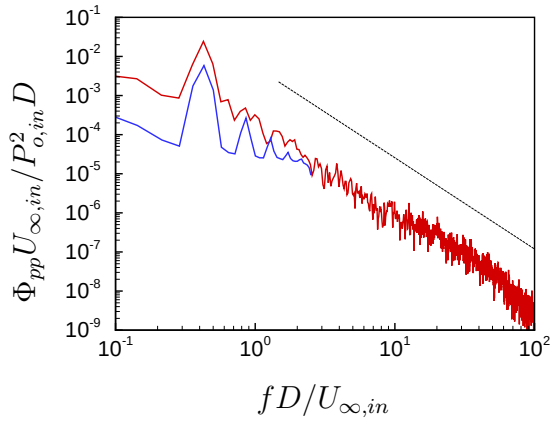
(b) K-3 probe ($\theta = 97.7^\circ$)



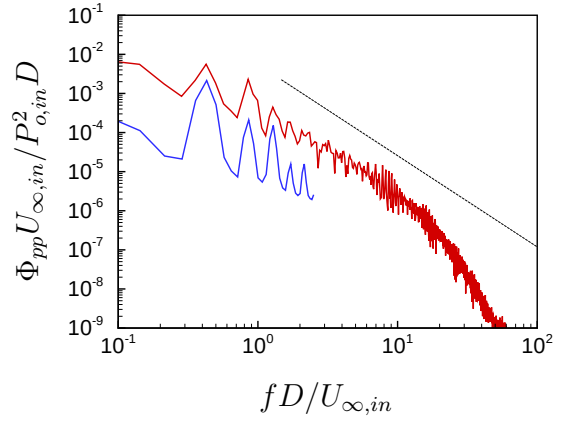
(c) K-5 probe ($\theta = 105^\circ$)



(d) K-7 probe ($\theta = 112.3^\circ$)



(e) K-9 probe ($\theta = 119.7^\circ$)



(f) K-10 probe ($x - x_t = 0.493D$)

Figure 5.10. Frequency spectra of pressure fluctuations at Kulite probe locations: —, numerical predictions; —, experimental measurements of case 4 by Vorobiev et al. [63]; - - - -, line with $-7/3$ slope.

is similar to that of the LES mesh. The total size of the beam grid is 7.67×10^6 grid points. It has 235 mesh points in the x' -direction, whereas the Shack-Hartmann wavefront sensor employed in the experiment has 60 subapertures in the same direction.

The optical results were calculated over a time period $T_{\text{stats}} = 18D/U_{\infty, \text{in}}$. The density field was collected every 10 time steps, equivalent to a sampling frequency of 2.7 MHz, whereas the sampling frequency of the 2-D wavefront sensor is 13.5 kHz.

At the beam grid points located outside the computational domain, the density was set to be unity. The effect of this approximation will be included in the steady lensing term, which is the time-averaged wavefront and is removed before studying optical distortions.

Figure 5.11(a), (b) and (c) display the time history of piston, tilt and tip components of the optical distortions, respectively, compared to the pressure fluctuations, $p'/P_{o, \text{in}}$, at the aperture center at $\theta = 105^\circ$. It is shown that the piston and tilt components are highly correlated with turret-surface pressure fluctuations, and both are strongly affected by the shock wandering. On the other hand, the spanwise tip component is an order of magnitude smaller than the streamwise tilt component and has no significant low-frequency content.

Figure 5.12 depicts an instantaneous piston-tip-tilt-removed optical wavefront. It is shown there is no significant variation of wavefront distortions in spanwise direction. Therefore OPD is spanwise-averaged hereafter to study its variation in the streamwise direction.

Figures 5.13(a) and (b) display the instantaneous spanwise-averaged OPD of the numerical simulation and the experimental measurements of case 4, respectively. Similar patterns are observed. The footprint of the periodic motion of the oblique shock over the aperture is clearly indicated by the discontinuity in the wavefront, which is more pronounced during the downstroke, and is weaker during the upstroke.

The effect of the oblique shock on optical distortions will be demonstrated in more details in Section 5.5.

5.5 Flow Field Analysis Using Phase Averaging

It is demonstrated in Section 5.4 that the piston and streamwise tilt components of the wavefront distortions are highly correlated with the turret-surface pressure fluctuations. Therefore, a phase averaging is employed to analyze the periodic flow features and the associated aero-optical distortions, and to demonstrate their relation with the shock oscillations. The purpose of this analysis is to provide relevant data for the design of adaptive-optical system to mitigate shock-induced optical distortions.

The implementation of phase averaging is explained for a given discrete signal, $s = [s_1, s_2, \dots, s_N]$, which has N samples collected at equally-spaced time instants. To perform the phase averaging, the signal is divided into a number of non-overlapped windows of the same time period T_{cycle} , which is corresponding to the fundamental frequency of the shock motion. Each window of the signal has the same number of n samples. Averaging is performed to calculate the phase-averaged signal as:

$$\hat{s}_i = \sum_{k=1}^{\mathcal{M}} s_{i+(k-1)n}, \quad i = 1, 2, \dots, n, \quad (5.7)$$

where $\hat{s}$ is the phase-averaged signal. $\mathcal{M}$ is the total number of non-overlapped windows used in the phase averaging and k is the index of each window.

Using the methodology described in Eq.(5.7), a number of flow and optical quantities are phase averaged including the pressure signals p which are collected at the locations of K-1 to K-9 probes, the pressure fluctuations obtained at the same locations, the piston and streamwise tilt components of the wavefront distortions, the piston-tilt-tip-removed OPD, the density and the density fluctuations, $\rho' = \rho - \bar{\rho}$, collected on the beam grid.

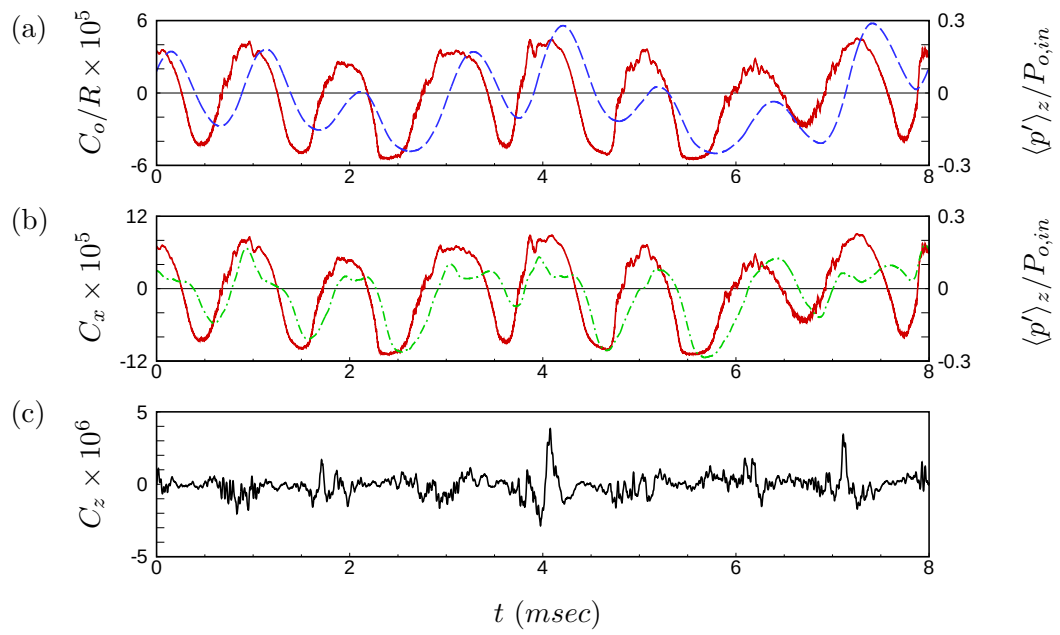


Figure 5.11. Instantaneous optical wavefront components and spanwise-averaged pressure fluctuations at the aperture center: — — —, piston (C_o/R); — · —, tilt (C_x); —, tip (C_z); —, spanwise averaged pressure fluctuations ($\langle p' \rangle_z / P_{o,in}$).

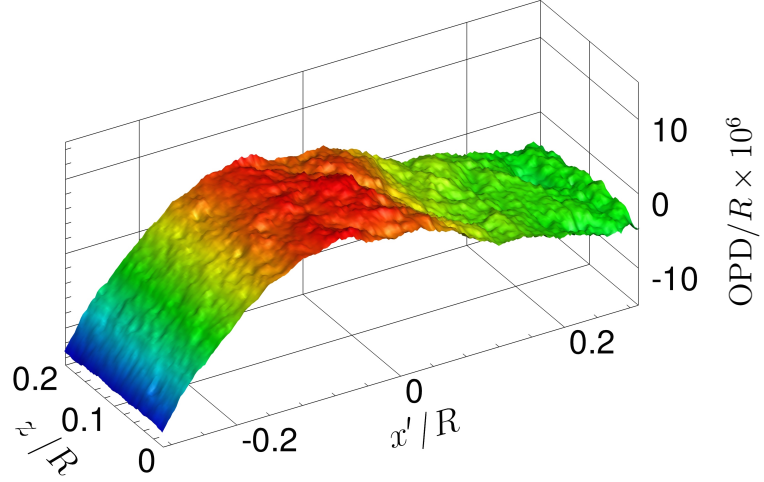


Figure 5.12. Instantaneous piston-tip-tilt-removed OPD.

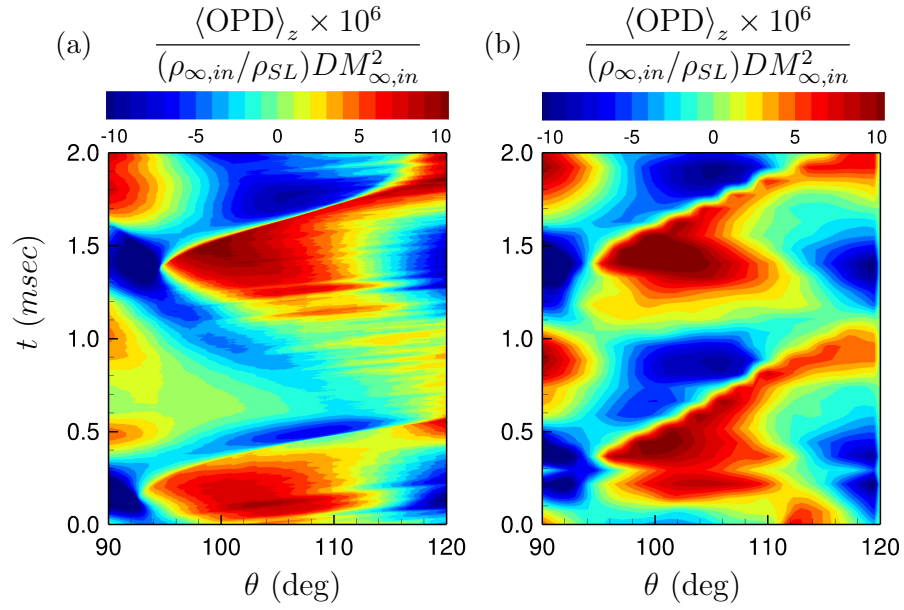


Figure 5.13. Instantaneous spanwise-averaged piston-tip-tilt-removed OPD: (a) numerical predictions; (b) experimental measurements in case 4 (Vorobiev et al. [63]).

The phase ϕ is detected using a reference sinusoidal function of the same fundamental frequency. The phase-and-spanwise-averaged pressure fluctuations at each of the K-probes from K-1 to K-9 and the reference signal are cross-correlated to determine the phase difference between the two signals. Then, the mean phase delay, $\Delta\phi_{\text{mean}}$, is calculated as an average over these cross-correlations of each of nine pressure fluctuations signals and the reference signal. The phases are corrected for all phase-averaged flow and optical quantities based on the calculated value of $\Delta\phi_{\text{mean}}$.

Figure 5.14 depicts the iso-contours of the phase-averaged turret-surface pressure fluctuations for a range of elevation angle from $\theta = 90^\circ$ to 120° covering the stream-wise aperture size, where the footprint of the periodic shock motion is unmissable.

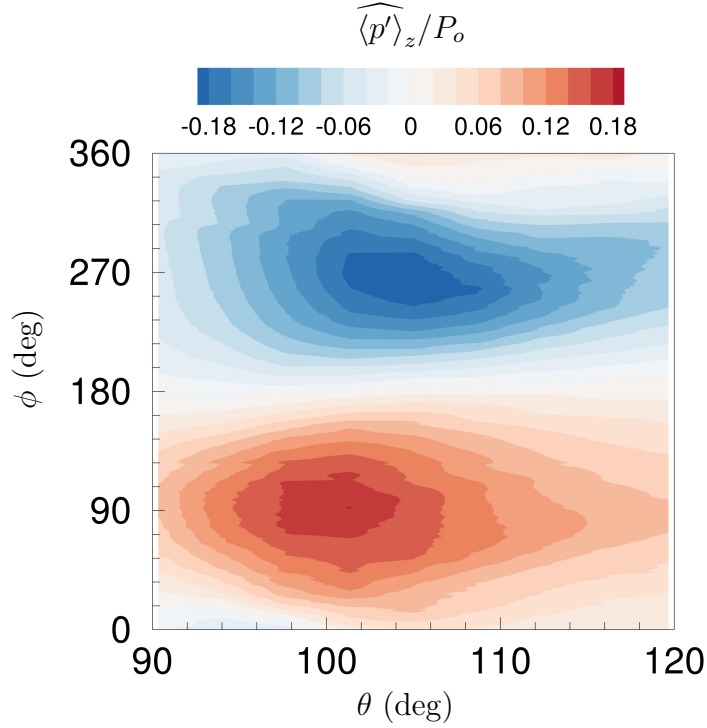


Figure 5.14. Phase-and-spanwise-averaged pressure fluctuations over the turret surface.

The phase-averaged piston and tilt are plotted in Figure 5.15 along with the phase-averaged turret-surface pressure signal at the aperture center (i.e. $\theta = 105^\circ$). The piston and tilt components are highly correlated with the surface pressure. The phase-averaged piston has a phase shift of $\Delta\phi = 60^\circ$ with respect to the phase-averaged pressure fluctuations at the aperture center. The positive peak of the tilt is achieved at the same phase $\phi \simeq 90^\circ$ as the pressure signal, whereas its negative peak is attained at a phase delay of $\Delta\phi = 30^\circ$ from the negative peak of the pressure fluctuations. The absolute value of the negative peak of tilt is approximately 50% higher than that of its positive peak, indicating that the time-averaged tilt is negative. This is due to the effect of the separated shear layer which enlarges downstream. The density field in the separation zone is lower, which causes the optical beam to propagate a longer distance with lower speed at the upstream portion of the aperture, resulting in a negative time-averaged streamwise tilt component of the wavefront distortions.

Figures 5.16(a) and (b) display the iso-contours of OPD as a function of elevation and phase angles from the numerical simulation and experimental measurements of case 4, respectively. Although both sets have similar wavefront structures, the magnitudes of optical distortions from LES is slightly lower than that of the experiment. The footprint of the shock oscillations above the aperture is noticeable in both maps. However, the discontinuities in the numerical results are weaker because the limited number of flow cycles considered in the phase averaging are not identical, and thus the phase averaging among these cycles smoothes the shock-induced discontinuity in the phase-averaged wavefront.

The temporal evolution of the flow oscillation and associated optical distortion is presented at six representative phases in Figures 5.17(a-f). The plot at the upper-left corner of each figure displays the phase-averaged piston and tilt components and pressure fluctuations at the aperture center, and the red dot on the pressure curve indicates the current phase. The plot at the lower-left corner of each figure depicts

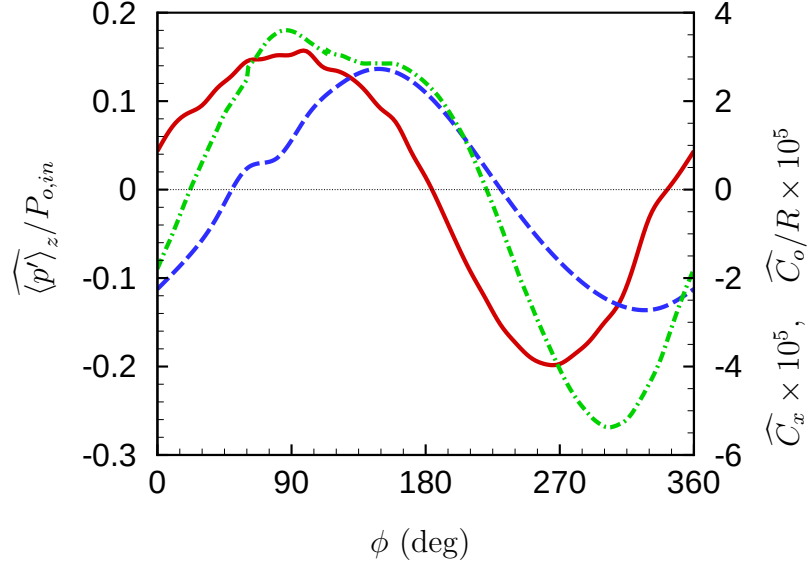


Figure 5.15. Phase averaged flow and aero-optical quantities: $---$, piston (C_o/R); $- \cdot -$, tilt (C_x); $—$, pressure fluctuations at the aperture center.

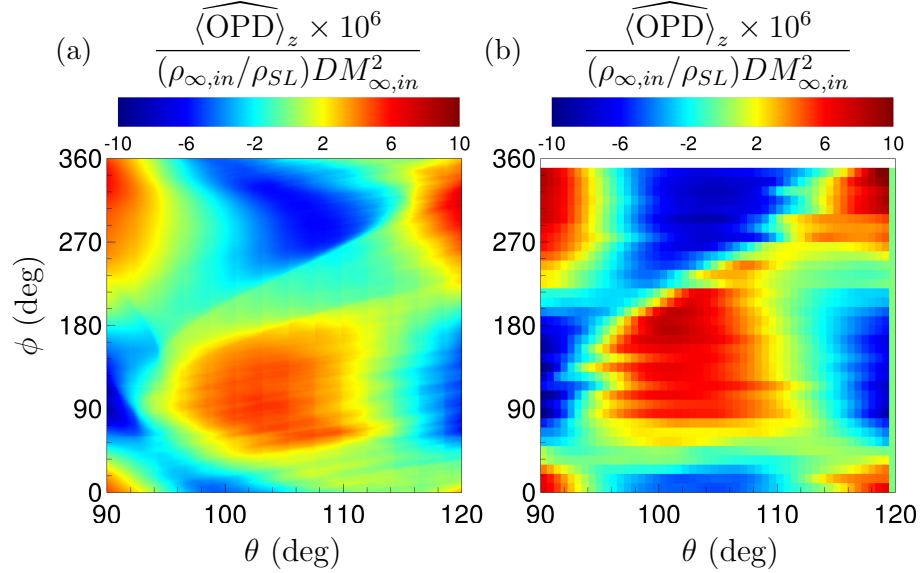


Figure 5.16. Normalized phase-and-spanwise averaged OPD: (a) numerical predictions; (b) experimental measurements of case 4 (Vorobiev et al. [63]).

the phase-and-spanwise-averaged spatial distribution of the OPD and turret-surface pressure at the K-probes in a range of elevation angle of $\theta = 90^\circ$ to 120° . The right hand side of each figure portrays the density field within the optical beam in three different forms. It displays, from left to right, instantaneous density at the midspan (ρ), phase-and-spanwise-averaged density and phase-and-spanwise-averaged density fluctuations, respectively.

At the phase $\phi = 0^\circ$, as shown in Figure 5.17(a), the shock strength is noticeable through the turret-surface pressure. Upstream the oblique shock, the density fluctuations have a negative sign due to the flow expansion, whereas they have a positive sign behind the shock because of the flow compression. Therefore, the field of density fluctuations has a negative spatial correlation across the shock due to the reversal of their sign. Consequently, The phase-averaged correlation length of OPD decreases across the shock, and the magnitude of the phase-averaged OPD for a travelling beam across the oblique shock will be reduced according to Sutton's linking equation (Eq.(3.7)). The spanwise-averaged OPD has no obvious streamwise change as demonstrated in the aperture-surface pressure because the discontinuity location of the oblique shock above the aperture has a variable elevation angle (θ) along the beam-propagation direction. And, the integration of density along the optical beam will fade out the discontinuity caused by the oblique shock.

At the phase $\phi = 90^\circ$, as shown in Figure 5.17(b), the turret-surface pressure fluctuations and tilt component reach their positive peak points. The shock approaches the top of the cylindrical turret. There is no significant jump in the streamwise distribution of the turret-surface pressure indicating the decrease of the shock strength. However, the shock-induced adverse pressure gradient is sufficiently high to induce the flow separation immediately downstream the shock.

Figure 5.17(c) portrays the flow evolution at phase $\phi = 150^\circ$. The oblique shock moves further upstream and leaves the domain of the optical beam. As a results,

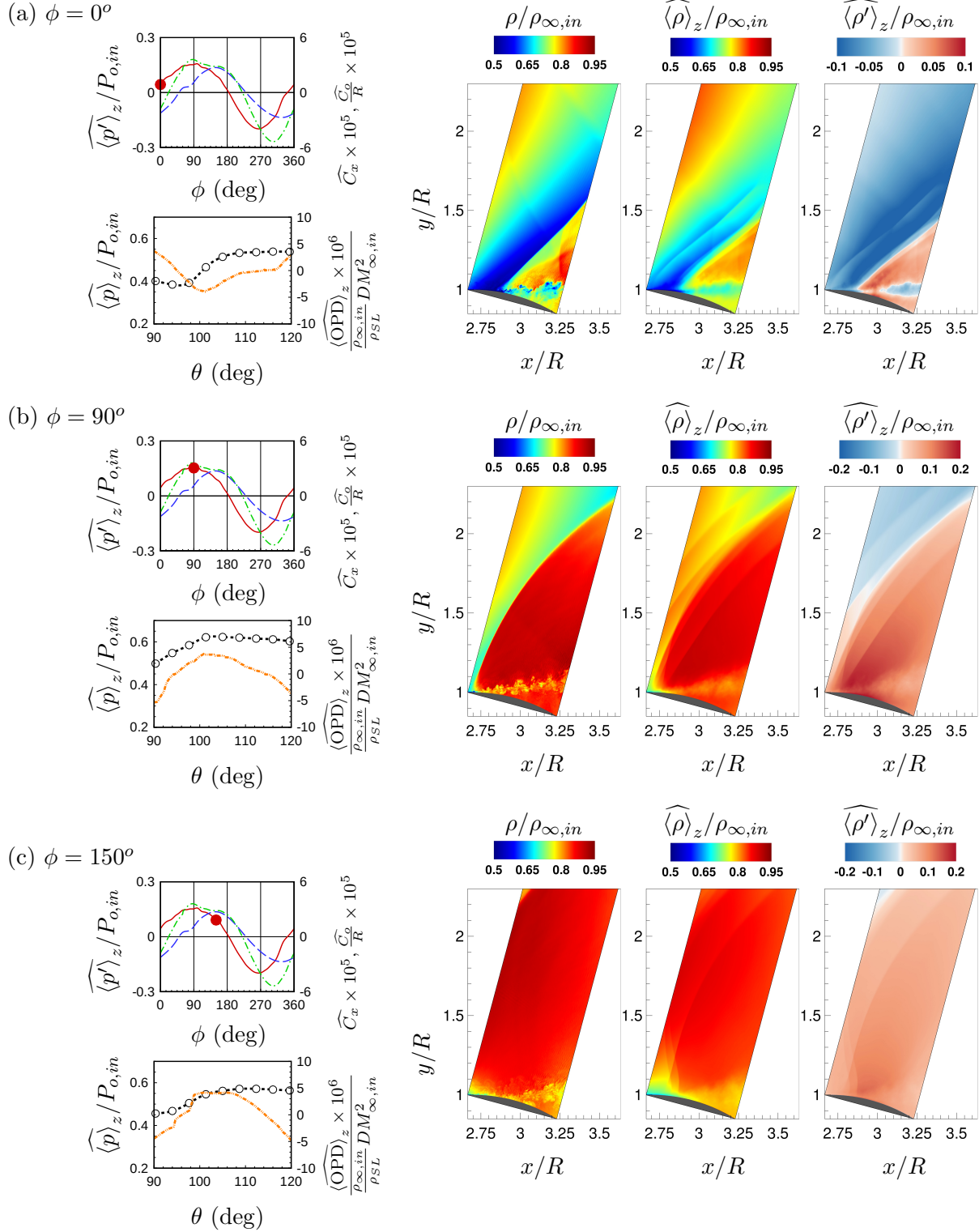


Figure 5.17. Flow evolution at representative phases. Right: contours of the instantaneous density, phase-and-spanwise-averaged density and density fluctuations on the beam grid. Left: phase-averaged pressure and optical distortions: — — —, piston; — · —, tilt; — — —, pressure fluctuations at $\theta = 105^\circ$; - - o - -, turret pressure at K-probes; — · —, OPD.

the density fluctuations have the same positive sign along the beam-propagation direction, which leads to buildup of the optical distortions and thus OPD at the aperture center approaches its positive peak at this phase. And, the piston component reaches its positive maximum as the phase-averaged density approaches its maximum value along the beam-propagation direction.

Figure 5.17(d) shows that the turret-surface pressure fluctuations approach their negative peak at the phase $\phi = 270^\circ$. The flow re-accelerates over the turret surface forming a small shock which leads to a sudden increase in the wavefront slice. A weak reflected shock from the previous cycle is moving upstream. There is a reversal in the sign of the density fluctuations across the reflected shock as previously discussed.

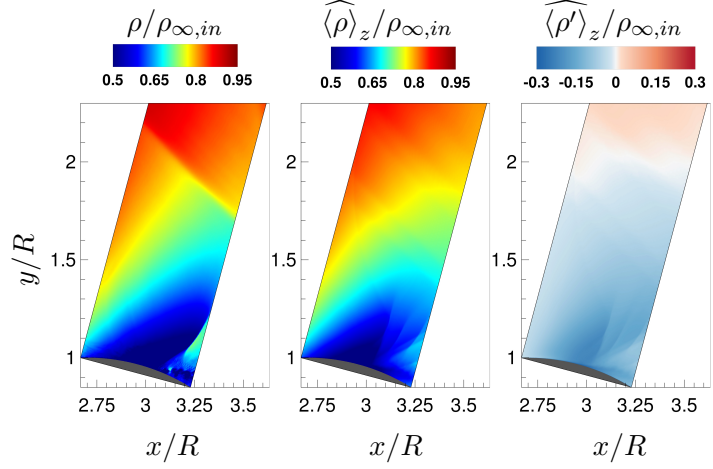
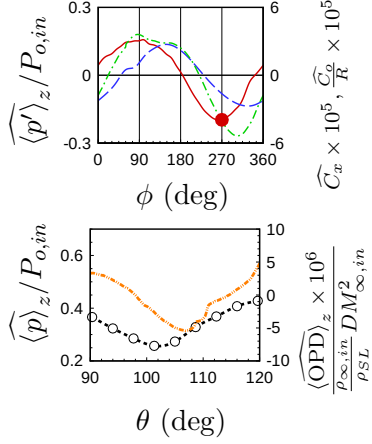
At the phase $\phi = 300^\circ$, as depicted in Figure 5.17(e), the shock strengthens during the upstroke. The tilt component and the piston-tip-tilt-removed OPD approach their negative peaks.

Figure 5.17(f) display the flow at phase $\phi = 330^\circ$, where the shock approaches its maximum strength during the upstroke and phase-averaged density along the beam-propagation direction approaches its minimum value. As a results, the piston component reaches its negative peak.

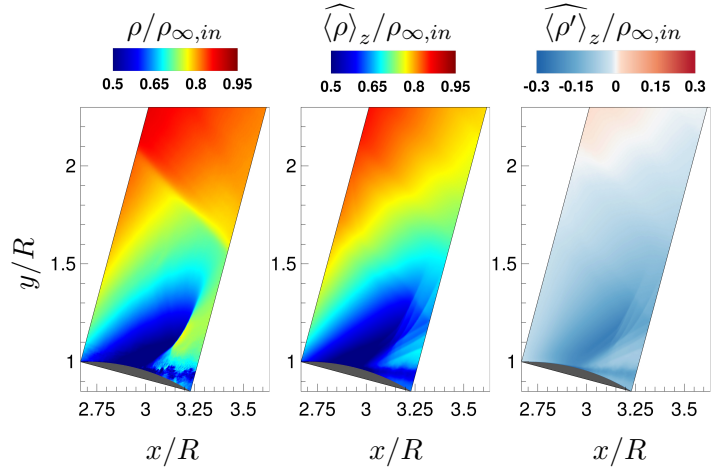
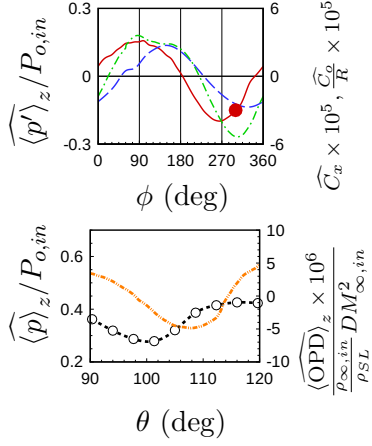
5.6 Mechanism of the Low-Frequency Shock Movement

The wall-modeled LES predicts large-scale shock oscillations consistent with the “transonic resonance” observed by Vorobiev et al. [63] in case 4. The three-dimensional flow visualizations in Figures 5.7(a-f) illustrate a strong coupling between the shock and flow separation. During the downstroke, the attached flow expands and a strong shockwave moves downstream leading to larger separation and a flow reversal. During the upstroke, the shock becomes weaker. These flow oscillations resemble the shock/boundary-layer interactions observed by Johnson and Papamoschou [30] in a set of experiments of over-expanded flow inside a 2-D divergent nozzle. Olson and

(d) $\phi = 270^\circ$



(e) $\phi = 300^\circ$



(f) $\phi = 330^\circ$

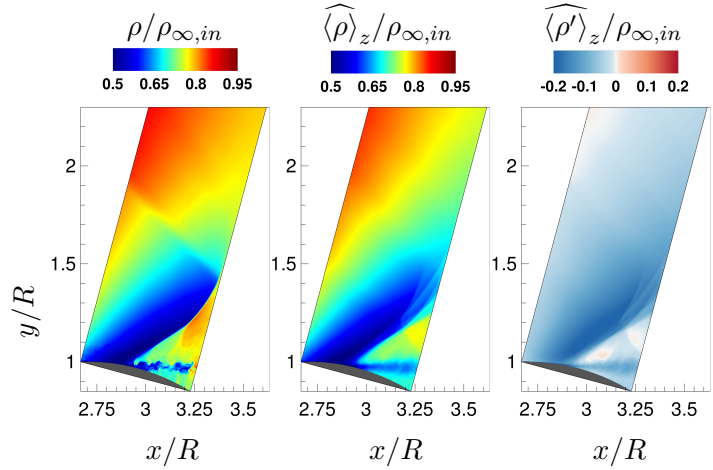
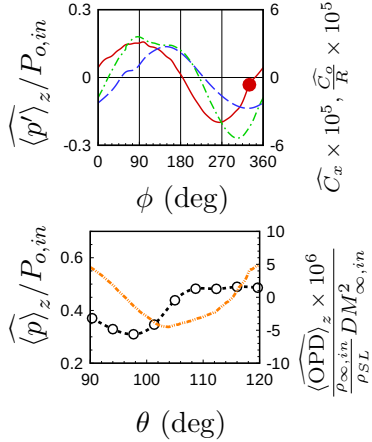


Figure 5.17 (Continued). Flow evolution at representative phases. Right: contours of the instantaneous density, phase-and-spanwise-averaged density and density fluctuations on the beam grid. Left: phase-averaged pressure and optical distortions: — — —, piston; - - - -, tilt; — — —, pressure fluctuations at $\theta = 105^\circ$; - - o - -, turret pressure at K-probes; — · — · —, OPD.

Lele [50] proposed a physical mechanism of the shock/boundary-layer interactions for over-expanded 2-D flow based on their simulation data generated by wall-resolved LES at a reduced Reynolds number.

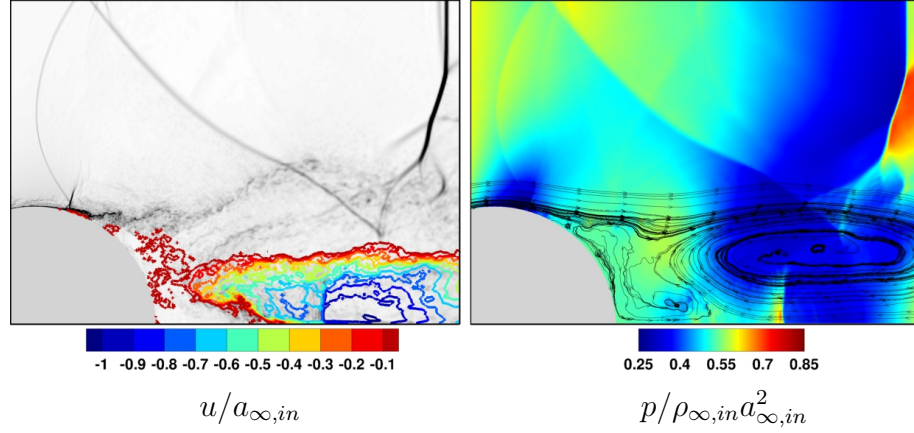
Inspired by the computational analysis of Olson and Lele [50] for over-expanded nozzle flow, the flow field from wall-modeled LES is employed for a more in-depth exploration of the underlying physics of shock-separation interactions. This mechanism is best understood by visualizing one period of the shock motion. Figures 5.18(a-f) portray the development of shockwave and separated shear layer at a sequence of events during one cycle. The left sub-plot in each figure visualizes the numerical schlieren, as defined in Eq.(5.6), along with colored iso-contours of instantaneous streamwise velocity of reversed flow regions, where $u < 0$. The right sub-plot in each figure portrays the corresponding contour map of the instantaneous pressure field accompanied by instantaneous streamlines to display the large-scale vortical structures.

Figure 5.18(a) depicts the start of the events sequence at time instant $t = 0$. The flow accelerates over the turret surface causing a local small shock to move downstream. The induced separated shear layer contains small-scale vortical structures.

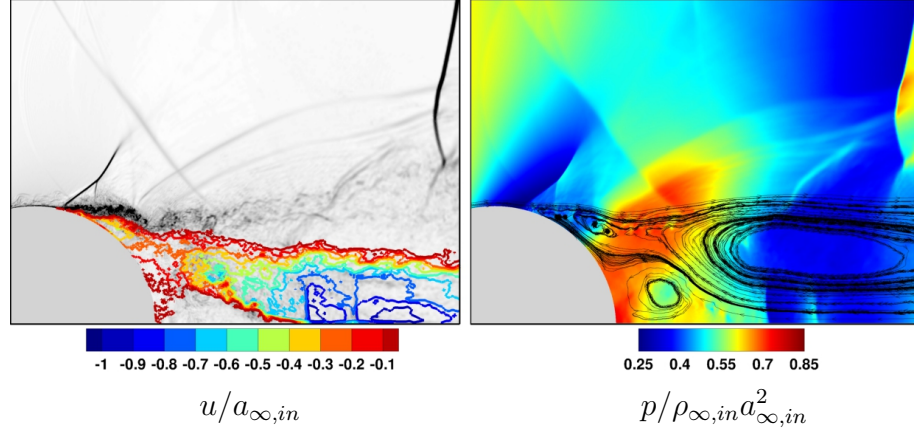
As the flow expands and the shock moves further downstream, the Mach number of the flow upstream the shock increases leading to stronger adverse pressure gradient. As a result, the shock-induced separation enlarges and the flow begins to reverse behind the shock, as displayed in Figure 5.18(b) at $t = 0.43D/U_{\infty,in}$.

The flow evolution from $t = 0.86D/U_{\infty,in}$ to $t = 1.29D/U_{\infty,in}$ is shown in Figure 5.18(c) and (d), where the reversed flow greatly thickens the separated shear layer. The vortex core in the wake is indicated by a low-pressure region. The mass and velocity influx of the reversed flow induced by the separation vortex leads to a pressure rise behind the shockwave, which consequently moves upstream to compensate for the pressure buildup downstream the shock, and its intensity is getting

(a) $t = 0$



(b) $t = 0.43D/U_{\infty,in}$



(c) $t = 0.86D/U_{\infty,in}$

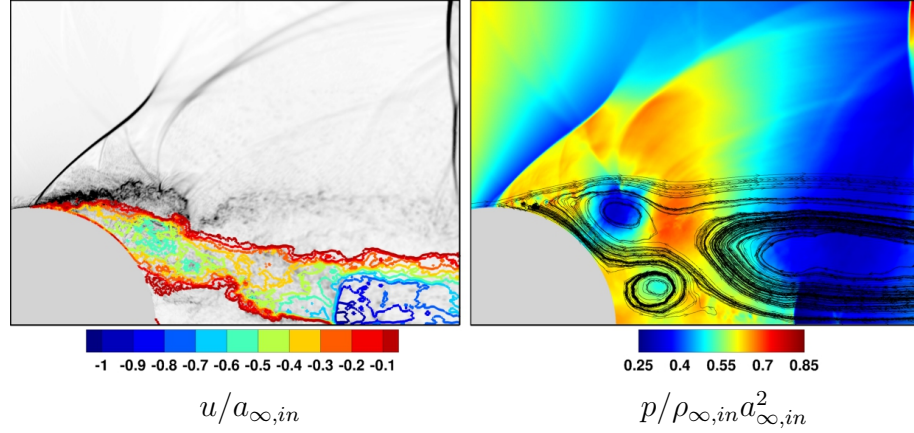
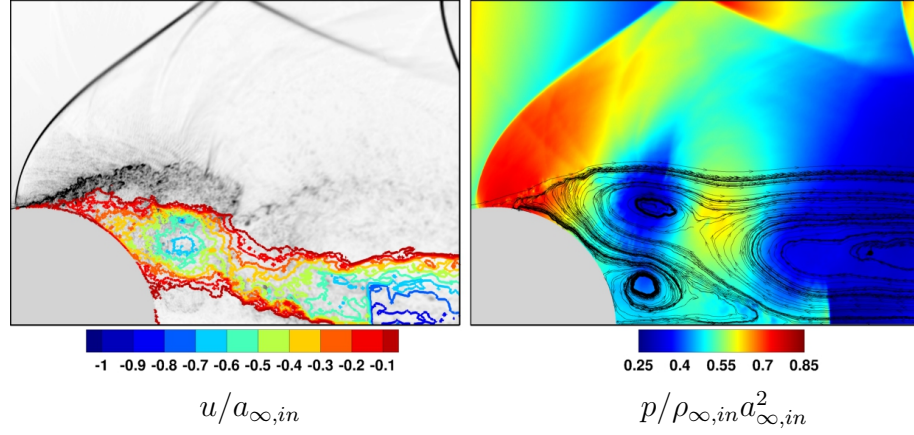
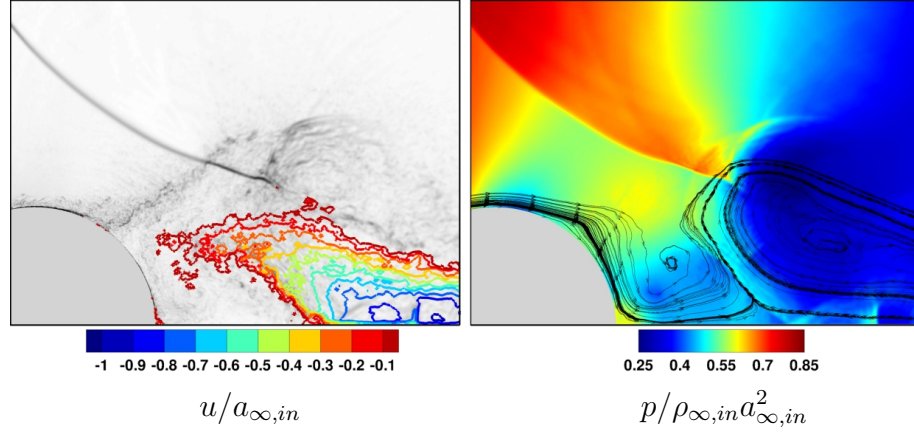


Figure 5.18. Development of shock-separation interaction at selected time instants: (left) numerical schlieren with iso-contours lines of streamwise velocity of reversed flows; (right) contour plot of pressure with instantaneous streamlines.

(d) $t = 1.29D/U_{\infty,in}$



(e) $t = 2.17D/U_{\infty,in}$



(f) $t = 2.35D/U_{\infty,in}$

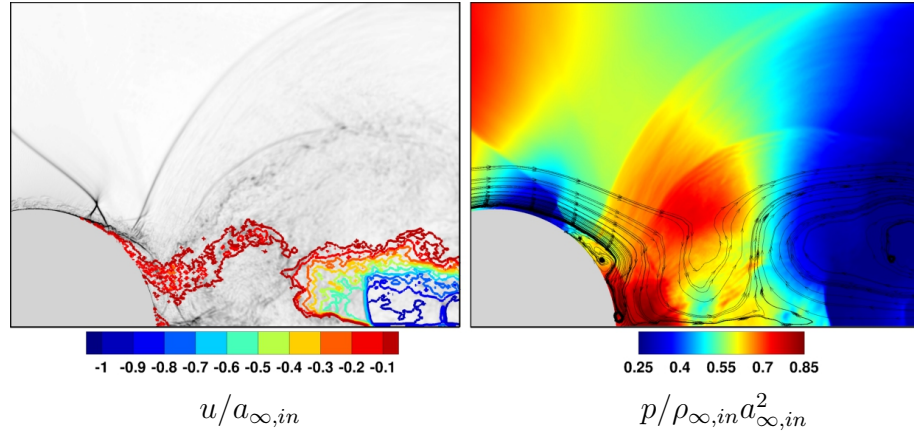


Figure 5.18 (Continued). Development of shock-separation interaction at selected time instants: (left) numerical schlieren with iso-contours lines of streamwise velocity of reversed flows; (right) contour plot of pressure with instantaneous streamlines.

weaker while moving upstream.

Figure 5.18(e) depicts the flow at $t = 2.17D/U_{\infty,in}$, where the shock is diminished upstream and the flow starts to expand again over the turret surface. The separated shear layer becomes thinner. And, there is no flow reversal over the turret surface. As a result, the flow re-accelerates again over the turret surface causing the shock to strengthen while moving downstream, and the cycle after $t = 2.35D/U_{\infty,in}$ as illustrated in Figure 5.18(f).

The aforementioned mechanism of the shock-separation interactions can be summarized below based on the selected events in one low-frequency cycle as shown in Figure 5.18(a-f):

- $tU_{\infty,in}/D = 0 - 0.43$: Shock moves downstream; separated shear layer thickens.
- $tU_{\infty,in}/D = 0.43 - 1.29$: Flow reversal occurs; the pressure behind the shock rises; shock moves upstream.
- $tU_{\infty,in}/D = 1.29 - 2.17$: Shock weakens when moving upstream; separation region shrinks.
- $tU_{\infty,in}/D = 2.17 - 2.35$: Flow accelerates over the turret surface; shock moves downstream; the cycle repeats.

During this cycle, the separated shear layer is strongly coupled with the shock position and strength through a time-delayed feedback mechanism. When the shock gets stronger while moving downstream, the separation bubble enlarges to push the shock upstream. And when the shock weakens during the upstroke, it shrinks to allow the shock to move downstream. There is a time delay between shock re-location and separated shear layer enlargement and contraction causing the shock cycle to be constantly repeated.

Vorobiev et al. [63] proposed a low-dimensional model to identify the global feedback mechanism. The oscillating shock system above the turret surface is described by a second-order ordinary differential equation of a simple harmonic oscillator (mass-spring system) with external forcing terms in terms of the separation pressure over

the turret surface (P_s):

$$\frac{d^2 P_s(t)}{dt^2} + \Omega^2 (P_s(t) - P_{back}) = C \frac{du_{ind}(t)}{dt}, \quad (5.8)$$

where Ω is the natural frequency of the system of the shock oscillations and P_{back} is the exit constant pressure. C is a constant. u_{ind} is the streamwise velocity which is induced by the separation vortical structure in the separation bubble and it is assumed to be positive towards the upstream direction. The separation-induced upstream-velocity represents the feedback forcing term affecting the oscillating system. The external forcing term should have the same natural frequency and be in phase with the system oscillations to achieve resonance.

Figure 5.19 depicts the time history of the pressure and the streamwise velocity at an elevation angle of $\theta = 97.7^\circ$ and at an off-wall distance of $2.5 \times 10^{-3} R$ from the turret-wall surface where the flowfield is relatively well-resolved. It is noted that the negative streamwise velocity fluctuations are in phase with the pressure fluctuations corresponding to the shock wandering over the turret. This points to the excitation of resonant feedback modes inside the separated shear layer as a potential source of the low-frequency shock oscillations in the transonic flow regime. The conclusion based on simulation data is consistent with the explanation from the proposed low-dimensional model Vorobiev et al. [63].

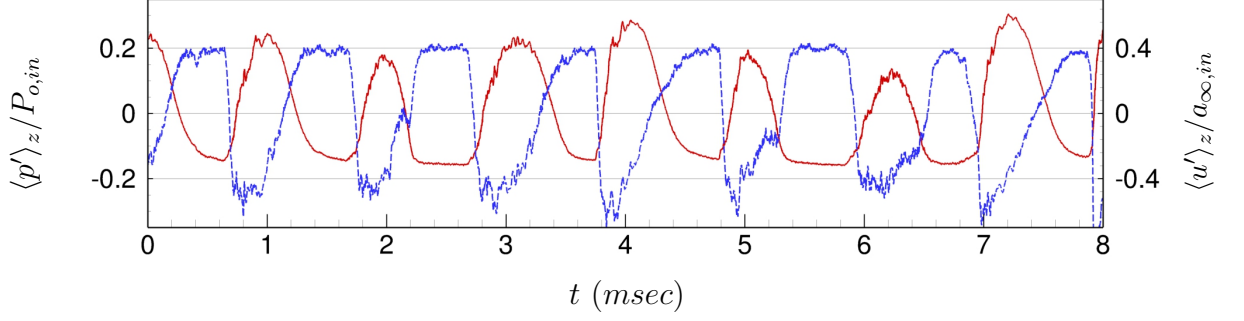


Figure 5.19. Time history of the pressure fluctuations and streamwise velocity fluctuations at $\theta = 97.7^\circ$ and an off-wall distance of $2.5 \times 10^{-3}R$ from the turret surface. —, streamwise-averaged pressure fluctuations; - - -, streamwise-averaged streamwise velocity fluctuations.

5.7 Concussions

Transonic flow over a cylindrical turret was computed using wall-modeled LES under the same flow conditions as in a test case 4 in the experiment of Vorobiev et al. [63]. In this test case, the flow is choked inside the test section, and the inflow is at $M_{\infty,in} = 0.68$ and Reynolds number $Re_{\theta,in} = 10^4$. The back pressure was determined via an iterative procedure to provide a best match with the mean pressure on the turret surface. The low-frequency shock oscillation is well-predicted by the numerical simulation and its fundamental frequency, $fD/U_{\infty,in} = 0.426$, agrees with the experimental measurements. However, the magnitude of pressure fluctuations is overpredicted, which may suggest the need for better grid resolution.

It is shown that the optical distortions are mainly affected by the low-frequency periodic motion of the shock over the optical aperture. The piston and streamwise tilt components of the wavefront distortions are highly correlated with the aperture-surface pressure fluctuations. The piston-tilt-tip-removed OPD has no significant variations in the spanwise direction. The patterns of wavefront distortions are consistent with the 2-D wavefront measurements in the experiment.

Phase averaging was performed for the periodic flow and optical quantities based on the fundamental frequency. The evolution of the flow and associated wavefront distortions are studied at representative phases in one cycle. The phase analysis demonstrates the effect of the oblique wandering shock above the aperture on the piston, tilt and high-order components of wavefront distortions.

Better understanding of the shock-separation interaction can aid in the development of novel flow-control strategies. The numerical data allow better understanding of the causes of the low-frequency shock oscillation over the turret surface in the transonic flow regime.

CHAPTER 6

SUMMARY AND OUTLOOK

6.1 Summary and Conclusions

Motivated by the demand for accurate and affordable numerical predictions of the aero-optical effects on airborne laser systems, LES with a wall model is evaluated systemtically in a set of canonical flow configurations including turbulent boundary layers and separated shear layer over a cylindrical turret at subsonic, transonic and supersonic Mach numbers. The numerical predictions are compared with experimental measurements and previous numerical results at low to moderate Reynolds numbers.

A low-dissipative unstructured finit-volume LES code, CharLES, developed by Cascade Technologies Inc., is employed in this research with the equilibrium stress-balance model. The code is augmented with capabilities required for this research, such as generation of turbulent boundary layers via the recycling-and-rescaling technique [40, 62] and collection of instantaneous flow variables on a secondary grid. The optical wavefront distortions are calculated by ray tracing based on the instantaneous density field from the LES and the Gladstone-Dale relation.

Turbulent boundary layers are simulated at Mach numbers $M_\infty = 0.5, 2.0, 3.0$ and 4.3 and Reynolds numbers ranging from $Re_\theta = 3.55 \times 10^3$ to 6.9×10^4 . A grid sensitivity analysis is conducted for low- and high-Reynolds-numbers boundary layers. It is found that predictions of density fluctuations and optical distortions are more demanding in terms of mesh resolution compared to velocity fluctuations. Reasonable

agreement is obtained between the numerical results and experimental measurements in terms of the magnitudes and structures of optical distortions. However, the simulations underpredict the decrease of $\overline{\text{OPD}}_{rms}$ with Mach number, and discrepancy with experimental data increases at high Mach numbers. The relationship between the length scales of wavefront distortions and density fluctuation structures in boundary layers is established. The two-point spatial correlations of density fluctuations in the outer region of the boundary layer and of the optical distortions are consistent with large-scale flow structures, whose characteristic length scale is comparable to the boundary-layer thickness, even though the streamwise velocity fluctuations have a much longer streamwise correlation length.

Subsonic and transonic flows over a cylindrical turret are computed under experimental conditions. The subsonic flow aero-optical results show agreement with the experimental data of Gordeyev et al. [19, 21] and the wall-resolved LES results of Wang and Wang [66] at a reduced Reynolds number. In the transonic case, a hybrid central/ENO scheme is employed for shock capturing. The numerical simulation predicts accurately the frequency of shock-induced pressure oscillations on the turret surface and wind-tunnel wall. The piston and tilt components of the optical distortions are highly correlated with the pressure oscillations on the aperture surface due to the shock motion, and the low-frequency shock wandering is clearly identified in the piston-tilt-tip-removed OPD. This strong relationship between aperture-surface pressure oscillations and optical distortions due to the oblique shock motion is relevant to the design of adaptive-optics systems for aero-optical mitigation. The strong coupling between the shockwave and separated shear layer is investigated through time instants during one cycle of the shock oscillations.

6.2 Prospects for Future Work

The research presented in this dissertation can be improved and extended in a number of directions as discussed below.

6.2.1 Mesh Convergence Study of Transonic Flow over a Cylindrical Turret

The levels of the pressure fluctuations on the turret surface and wind-tunnel test section are over-predicted compared to the experiment. One possible reason is insufficient mesh resolution. The transonic flow simulation over a cylindrical turret described in chapter 5 has only been performed on a single grid. The grid sensitivity of flow variables and aero-optical quantities should be examined through a multi-levels mesh convergence study, as in the case of turbulent boundary layers (Chapter 3) and subsonic flow over a cylindrical turret (Chapter 4).

6.2.2 Transonic Flow over Hemispherical-Shaped Turrets

The numerical study of the transonic choked flow over a cylindrical turret in a wind-tunnel established the computational framework that can be extended to high-Reynolds-number flows over 3-D hemispherical or hemisphere-on-cylinder turrets at transonic flight speeds involving moving shocks. The numerical results of this research and the work of Bermejo-Moreno et al. [4] demonstrate that LES with sufficient mesh resolution in the outer region and an equilibrium wall model acting in less than 10% of the local boundary-layer thickness can predict flows with shock-induced separation. The numerical simulation of external flows over a 3-D turret will have more clearly defined boundary conditions compared to the internal choked flow over a cylindrical turret with unknown value of back pressure as described in Chapter 5. However, the flow over a hemispherical turret will be computationally much more expensive as the shockwave must be resolved long distance away from the turret in order to have accurate predictions of optical distortions.

6.2.3 Aero-optical Effects of Flows over Shallow Bumped Surfaces

It is known that the levels of optical distortions caused by separated shear layers (Chapter 4) are much higher compared to that caused by turbulent boundary layers (Chapter 3) due to the presense of large coherent vortical structures in the separated shear layer. One possible way to mitigate the optical distortions is by reducing the length scale of vortical structures through passive flow control devices [19]. Another potential solution is to replace the conventional turrets with shallow streamlined bumped surfaces, such as a 3-D version of the NASA hump, although it will reduce the field of regard. With such a hump, the flow is attached over the favorable-pressure-gradient surface and produce only a weak separation on the adverse-pressure-gradient side. A basic study of the aero-optical effects of turbulent boundary layers with pressure gradients and weak separation can be the first step.

6.2.4 Wall-Modeled LES for High-Mach-Number Supersonic Boundary Layers

It has been shown in Chapter 3 that the levels of optical distortions are overpredicted at high free-stream Mach numbers (e.g. $M_\infty = 3$ and 4.3) compared to the experimental results. Additional work is required to investigate the potential causes of this overprediction, to identify limitations of the wall modeling approach and to develop improved methods for accurate aero-optical predictions.

APPENDIX A

RELATION BETWEEN SPATIAL CORRELATIONS OF OPTICAL PATH DIFFERENCE AND DENSITY FLUCTUATIONS

If an optical wavefront traverses a turbulent boundary layer from the bottom wall in the cross-flow direction and local homogenous turbulence is assumed, the streamwise spatial correlation of optical-path-difference (OPD) at a given off-wall location ($L_{y'}$) is define as:

$$\mathcal{R}_{\text{OPD}}(\Delta x) = \frac{\left\langle \overline{\text{OPD}(x, z, t) \text{OPD}(x + \Delta x, z, t)} \right\rangle}{\left\langle \overline{\text{OPD}^2(x, z, t)} \right\rangle}, \quad (\text{A.1})$$

where $\langle \quad \rangle$ notates for two-dimensional spatial averaging in the streamwise (x) and spanwise (z) directions. Time-averaging is notated by the overline ($\overline{\quad}$).

By substitution from Eq. 1.4 in Eq. 1.3, the optical-path-length (OPL) at an off-wall location, $L_{y'}$, is rewritten as:

$$\text{OPL}(x, z) = L_{y'} + K_{GD} \int_0^{L_{y'}} \rho(x, y, z, t) dy. \quad (\text{A.2})$$

By decomposing $\rho(x, y, z, t) = \bar{\rho}(x, y, z) + \rho'(x, y, z, t)$, the above equation will be rewritten as follows:

$$\text{OPL}(x, z, t) = L_{y'} + K_{GD} \int_0^{L_{y'}} \bar{\rho}(x, y, z) dy + K_{GD} \int_0^{L_{y'}} \rho'(x, y, z, t) dy \quad (\text{A.3})$$

Since, the optical-path-difference (OPD) *without* tilt-tip removal is defined in Equation 1.5. It be rewritten in terms of density fluctuations as:

$$\text{OPD}(x, z, t) = K_{GD} \int_0^{L_{y'}} \rho'(x, y, z, t) dy. \quad (\text{A.4})$$

By substitution Eq. A.4 in Eq. A.1, the spatial streamwise correlation of OPD is defined in terms of density fluctuations as:

$$\mathcal{R}_{\text{OPD}}(\Delta x) = \frac{\left\langle \overline{\int_0^{L_{y'}} \rho'(x, y_1, z, t) dy_1 \int_0^{L_{y'}} \rho'(x + \Delta x, y_2, z, t) dy_2} \right\rangle}{\left\langle \left(\int_0^{L_{y'}} \rho'(x, y, z, t) dy \right)^2 \right\rangle}. \quad (\text{A.5})$$

Let $y = y_1$ and $\Delta y = y_2 - y_1$. Therefore, $dy_1 = dy_2$ and $d\Delta y = d\Delta y_2$. And Eq. A.5 is reformulated as:

$$\mathcal{R}_{\text{OPD}}(\Delta x) = \frac{\int_0^{L_{y'}} \int_{-y}^{L_{y'}-y} \left\langle \overline{\rho'(x, y, z, t) \rho'(x + \Delta x, y + \Delta y, z, t)} \right\rangle d\Delta y dy}{\left\langle \left(\int_0^{L_{y'}} \rho'(x, y, z, t) dy \right)^2 \right\rangle}. \quad (\text{A.6})$$

Since the 2-D spatial correlation of density fluctuations (ρ') in the streamwise and wall-normal directions is written as:

$$\mathcal{R}_{\rho\rho}(\Delta x, \Delta y; y) = \frac{\left\langle \overline{\rho'(x, y, z, t) \rho'(x + \Delta x, y + \Delta y, z, t)} \right\rangle}{\left\langle \overline{\rho^2(x, y, z, t)} \right\rangle}, \quad (\text{A.7})$$

From Eq. A.6 and A.6, the spatial correlation of OPD can be written in terms of the

spatial correlation of density fluctuations as follows:

$$\mathcal{R}_{\text{OPD}}(\Delta x) = \frac{\int_0^{L_{y'}} \int_{-y}^{L_{y'}-y} \left\langle \overline{\rho^2(x, y, z, t)} \right\rangle \mathcal{R}_{\rho\rho}(\Delta x, \Delta y) d\Delta y \, dy}{\left\langle \left(\int_0^{L_{y'}} \rho'(x, y, z, t) dy \right)^2 \right\rangle}. \quad (\text{A.8})$$

By substituting Eq. A.8 in Eq. 3.4, the streamwise correlation length of OPD is written in terms of the density field spatial correlation:

$$\Lambda_{\text{OPD},x} = \frac{\int_0^{L_{y'}} \left\langle \overline{\rho'^2(x, y, z, t)} \right\rangle \int_{-y}^{L_{y'}-y} \int_{\Delta x_{\min}}^{\Delta x_{\max}} \mathcal{R}_{\rho\rho}(\Delta x, \Delta y; y) d\Delta x \, d\Delta y \, dy}{2 \left\langle \left(\int_0^{L_{y'}} \rho'(x, y, z, t) \, dy \right)^2 \right\rangle}. \quad (\text{A.9})$$

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